"The Importance of Working for Earnest": Climbing the Job Ladder through Firms' Connectivity*

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Abstract. Do workers consider a firm's "springboard" value in terms of future job opportunities when choosing an employer? Using a search model of the labor market, I introduce the idea that firms differ in enhancing their employees' chances of receiving external job offers. The model informs a firm-level proxy for outside job offers received by workers. This measure empirically aligns with key model predictions: 1) it negatively correlates with both firm-specific tenure and young workers' entry salaries, revealing a compensating differential; and 2) it suggests that workers enjoy a salary premium upon leaving such firms, indicative of faster career progression. The model is estimated on administrative data from Italy and successfully captures key aspects of labor market dynamics. My channel explains 10% of the overall job-to-job transitions and shows how firm-induced variation in job search can be a significant driver of inequality, especially at the bottom of the wage distribution.

JEL-Classification: J31, J24, J62, E24.

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1 — Introduction

Firms' non-monetary characteristics affect workers' mobility decisions and career outcomes (Sorkin, 2018). A growing body of recent research has sought to understand how these traits offer compensating differentials to workers. Many of these characteristics relate to *current* opportunities, such as alternative payment schemes (Card et al., 2018), variations in training and human capital accumulation rates (Gregory, 2023; Arellano-Bover and Saltiel, 2021), or differences in job security (Jarosch, 2023). Other characteristics are instead related to *future* opportunities, such as the likelihood of securing higher-paying jobs later in one's career. The ability of workers to take into account the potential for receiving more future job opportunities when selecting their employers represents a significant yet understudied labor market mechanism, which is the focus of this paper.

Particularly for younger workers, the choice of early-career employers alongside this non-monetary characteristic is pivotal as it impacts their current working conditions and future wages and opportunities. Not all workers join firms with the intention of long-term tenure; some prioritize how placements enable future job prospects, acting as "springboards" in the labor market. Understanding the role of today's workplaces in facilitating better opportunities in the future is thus crucial. Indeed, it informs how workers make their occupational choices and how these choices influence labor market frictions, a recognized contributor to earnings inequality (Hornstein et al., 2011).

Overview. This paper explores the role of firm-specific differences in providing employees with better external job opportunities. Delving into employees' decision-making in choosing employers, I focus on how firms' unique contributions to career advancement trade off against immediate salary. I propose a novel intuition: that firms are heterogeneous in their effectiveness at increasing workers' likelihood of receiving offers from other firms. This so far unexplored heterogeneity influences the movement choices of employees in the labor market. This perspective shifts the heterogeneity in search behavior from workers to firms, challenging the traditional view that attributes it to variation in workers' efforts. Instead, my research underscores the crucial role of the current employer in offering diverse career growth opportunities to its employees.

I build a search model of the labor market that includes heterogeneous workers and firms, where the search frictions workers face are influenced by their current employer. This formalizes the intuition that the employer's characteristics matter in determining the likelihood of an employee receiving job offers from other companies. Next, I present empirical evidence that supports the primary predictions of this model. This evidence leverages a unique matched employer-employee dataset provided by the Italian Social Security Institute (INPS). I link the model's primitive governing the likelihood of a worker receiving an external job offer to the firm's degree centrality in the job-to-job network.[1] With the size-adjusted degree proxying for outside offers, I show that employees tend to have shorter tenures at firms with higher centrality. Additionally, younger workers are more inclined to accept lower starting salaries at firms with higher centrality. This behavior indicates the existence of a *compensating differential*, which I demonstrate is particularly relevant for young and inexperienced workers. Furthermore, I show that there is a *premium* associated with this channel: workers who leave firms that act as "springboards" into better opportunities earn, on average, about 6% more within a year of leaving, compared to those who depart from more typical firms. I then estimate the model on the INPS data, confirming that it accurately reflects key dynamics of the labor market. I use it to analyze the efficiency of the proposed job ladder, highlighting that workers—especially those at the lower end of the wage distribution—often underestimate the importance of the firm's contribution in providing external offers. Additionally, through a counterfactual analysis, I show that the firms' heterogeneity in providing external job offers accounts for 10% of the observed job changes between firms. Moreover, it explains 15% of the wage inequality at the bottom of the distribution, as my channel contributes to the formation of pockets of low-productivity firms, which workers find challenging to exit.

Model. I start this work by building a search model of the labor market that incorporates heterogeneous workers and firms. The essential element of this framework is that firms vary along two dimensions: productivity and connectivity. *Productivity* in this context determines the combined output of a worker-firm match, while *connectivity* influences the likelihood of an employed worker receiving external job offers. In other words, this parameter dictates how employment at different firms results in varying rates of external offer arrivals. Workers who are searching for jobs while employed thus face a trade-off between productivity, which offers higher immediate wages, and connectivity, which can facilitate a faster progression up the job ladder. This dynamic may lead workers to consider positions at lower-productivity firms

1. In such a network, firms are nodes, and workers' movements between these firms are links.

if these jobs significantly enhance their chances of receiving new offers. This way, firms can offer differential compensation by reducing future search frictions for their employees. While the model does not explicitly incorporate a life-cycle dynamic, the connectivity value is higher for younger workers. This is because the connectivity value diminishes as employees ascend the job ladder (and thus become older), transitioning into increasingly productive firms. The model incorporates a wage equation that explicitly correlates wages with job values across the two dimensions of firm heterogeneity. The influence of a firm's productivity on worker compensation is ambiguous, depending on the bargaining power of the employees. However, connectivity acts as a clear compensating differential, with entry wages tending to decrease as a firm's connectivity increases.

Connectivity is conceptualized as a firm's ability to facilitate the movement of employees to and from various other firms. This concept can be interpreted through several plausible mechanisms. For example, firms may reduce search frictions via their input-output relationships or through connections with past co-workers (Caldwell and Harmon, 2019). A firm's perceived "prestige" in the labor market can be an important driver, as workers can use it to signal their ability. Additionally, corporate practices like transition assistance programs, where former employees become potential clients, also enhance a firm's connectivity. Moreover, differences in screening and hiring practices can result in some firms consistently providing better workers, increasing the probability of their employees being approached by other firms. This paper, however, does not directly delve into the specific mechanisms underlying the differences among firms in reducing search frictions. The primary focus is to examine the implications of such heterogeneity, rather than exploring its origins in depth.

Empirical measure and model's predictions. I connect the predictions of the model to empirical observations by linking a firm's connectivity parameter with its expected degree centrality in a job-to-job network. This network consists of nodes that represent firms that have had worker transitions in a given period, and the directed links indicate the movements of workers to and from these firms. I employ a rich administrative employer-employee dataset from Italy's private sector to construct a network of white-collar employees moving between sizable firms from 2008 to 2020. Out-degree centrality, defined as the number of distinct employers to which a firm's workers have relocated within the period, provides a local measure of a firm's role in mediating worker flows. By normalizing this measure for the firm's average size throughout the decade, I derive a per-worker out-degree centrality. This prevents the undue emphasis on larger firms, which may appear central simply due to their size, and offers an estimation of the external job opportunities an average employee encounters while employed at a firm.

Initially, I use this measure to show that firms with a higher per-worker out-degree are typically found in high-value-added service sectors. This metric correlates with productivity and profitability, indicating that firms acting as hubs for workers are financially strong. Then, I apply it to validate three reduced-form predictions of the model.

First, I show that workers spend shorter tenures in firms with higher per-worker degree. In particular, a 10% increase in the log out-centrality of the firm is associated with a 1.1% decrease in firm-specific tenure. This is a straightforward consequence of the core mechanism in the model, as per the increased likelihood of finding a better match, and it is directly present in the data.

Second, given the network structure, I use between-firm movements between 2018 and 2020 to show that young workers are willing to accept lower initial earnings in exchange for higher firm centrality. Specifically, the log salary in the first 6 months is negatively correlated with the firm's centrality: a 10% increase in the latter is associated with a 1.5% decrease in the former. I interpret this empirical finding as evidence of the compensating differential implied by my model. Workers accept lower salaries to access firms with higher degree in the job network, as they offer superior future career opportunities. I show that this negative relationship is driven by young, inexperienced workers, and does not apply to more mature employees. This is because the longer the labor market tenure, the higher the probability of workers being matched with productive firms, which pay higher salaries. For them, the opportunity to receive new job offers becomes less critical compared to newcomers. This is another direct implication of the job ladder structure of my model, which is again clearly present in the data.

Third, I show that workers transitioning out of highly central firms achieve higher earnings on average. I divide the sample of firms into two groups using an unsupervised clustering algorithm based on the centrality measures that account for employee inflows and outflows.[2] This approach ensures that the splitting procedure is entirely data-driven. I then compare workers leaving "springboard" firms (those with higher centrality, approximately 12% of the sample of firms) with workers leaving regular firms in an *event study* around a job-to-job

2. I use a k-means algorithm to split firms into two groups based on different possible measures of degree centrality in the job-to-job network.

movement. On average, the former group earns 6% more than the latter group one year after leaving the firm, controlling for individual- and firm-specific heterogeneity. I interpret these results as evidence of another key mechanism implied by my model: workers use higher connectivity firms to climb the job ladder faster. Employees who pass through these firms are thus expected to earn more due to the increased likelihood of being matched with a higher-productivity firm, compared to those who transition through less connected employers.

I further explore the contributions of leaving more central firms to employee earnings. I focus on understanding whether these effects stem from human capital accumulation or signaling mechanisms. To extend traditional views that typically consider education as a signaling device, I propose that previous employment experiences can similarly act as signals of a worker's quality. I leverage a simple intuition to decompose a past firm's impact on a worker's current wages into these two distinct parts—human capital accumulation and signaling: the contribution of the former depends on firm-specific tenure length, which is not true for the latter. I estimate an AKM (Abowd et al., 1999) model that includes an origin and destination firm's fixed effect, interacting the former with workers' tenure at the firm. This model allows for distinguishing between the impacts of short-term and long-term tenures at a past firm on current wages. These impacts are then attributed to either human capital accumulation or signaling effects. Firm effects related to signaling are positively correlated with a firm's centrality in the job network, while no such correlation exists for effects associated with longer tenures. I interpret this as evidence that centrally positioned firms contribute primarily through signaling mechanisms.

Structural estimation and counterfactuals. I then estimate the model on the same 12-year panel from INPS used for the reduced-form analysis, via indirect inference. Identification comes from three key sets of moments or reduced-form parameters that inform each main ingredient of the model: individual ability, firms' productivity, and firm's connectivity. In particular, I rely on cross-sectional features of the wage distribution, moments related to job changes, and the distribution of per-worker in- and out-degree in the job-to-job network induced by the model.

The estimated model captures these target metrics and implies plausible labor market dynamics and wage dispersion. I employ it for two primary exercises. First, I explore the efficiency characteristics of my framework, which includes standard search externalities due to workers' underestimation of the full value of future gains from on-the-job search. Similarly to Jarosch (2023), these externalities manifest in how workers rank firms' attributes, often trading off productivity and connectivity as they ascend the job ladder. In my setting, this leads to a consistent undervaluation of the role of connectivity, especially for workers at the lower end of the wage distribution, who frequently settle for less-than-optimal levels of compensating productivity. My analysis uncovers that these low-productivity, low-connectivity "pockets" contribute significantly to labor market inefficiencies, particularly affecting workers at the lower end of the wage distribution.

Second, I measure the importance of the heterogeneity in connectivity by shutting down this channel in a counterfactual exercise. In particular, I shut down the primary source of heterogeneity in the original model, the connectivity parameter, simplifying it into a "standard" labor market framework where all employed workers have a uniform probability of receiving a job offer. The analysis reveals that eliminating this heterogeneity leads to a 10% reduction in job-to-job transitions and significantly alters the distribution of wage changes. Specifically, in the counterfactual setting, transitions are more likely to come with larger wage increases, highlighting the pivotal role of connectivity in determining compensating differentials for workers. Additionally, the counterfactual model yields insights into wage distribution and job-matching dynamics. It exhibits a consistent underestimation of wage inequality, particularly at the lower end of the distribution. This underscores the model's capacity to generate "inequality pockets," where workers are stuck in low-productivity, low-connectivity roles. In a homogeneous labor market, these pockets are less prevalent, leading to a more evenly distributed set of job transition opportunities. This change may have particular implications for workers in the lower half of the wage distribution, potentially increasing their likelihood of securing better job offers.

Related literature. The paper contributes to several areas of literature, specifically those concerning heterogeneity in labor market outcomes, worker dynamics across firms, and optimal search behavior. To my knowledge, this is the first study to explicitly consider the heterogeneity of firms' contributions to future job opportunities, utilizing comprehensive administrative data on private-sector contracts in a large country.

A considerable body of research has examined the impact of firm-specific characteristics on labor market outcomes and how different employers influence workers' wages and careers.

Existing studies, such as those by Andersson et al. (2012), Card et al. (2013), Card et al. (2018), and Song et al. (2019) rely on the exogenous-mobility approach pioneered by Abowd et al. (1999), using matched employer-employee data. These papers primarily focus on characteristics affecting the current worker-firm relationship. In contrast, my framework highlights a channel that specifically impacts a worker's future value. Abowd et al. (2018), Bonhomme et al. (2019) and Di Addario et al. (2023) address employment at heterogeneous firms within dynamic frameworks, thereby connecting past and present employers, but do not consider heterogeneity in firms' ability to provide external job offers.

Several recent papers addressing the labor market effects of firm differences do so by incorporating a combination of search and human capital accumulation, as in the seminal work by Bagger et al. (2014). Among these, Gregory (2023); Wang (2021) and Jarosch (2023) align to my setting by involving a search model to address the role of specific firms' characteristics in explaining labor market dynamics. Other papers (Arellano-Bover and Saltiel, 2021; Arellano-Bover, 2022) adopted a reduced-form approach to similarly study the careers' effect of specific heterogeneity in employers. The importance of workplace characteristics also aligns with De La Roca and Puga (2017), who explore channels that could potentially account for the correlation between salaries and city size across regions. However, none of these papers either explicitly or implicitly incorporate a connectivity mechanism to explain past-to-present firm relationships, nor do they structurally link firms' characteristics in reducing search frictions for their employees via an increased probability of receiving outside offers. Moreover, I neglect human capital accumulation, focusing on a purely firm-driven mechanism, suggesting a larger role for signaling effects on the labor market rather than employer-specific training.

The concept of identifying significant compensating differentials explaining wage cuts upon job-to-job transitions is shared by a considerable number of recent papers, such as Nunn (2012), Sullivan and To (2014), Hall and Mueller (2018), Taber and Vejlin (2020), and Caplin et al. (2022). In particular, Sorkin (2018) adopts a revealed-preferences approach, exploiting a centrality measure on the job-to-job network to assess the importance of compensating differentials in workers' mobility behavior. In addition to this latter work, others like Nimczik (2020) and Huitfeldt et al. (2021) are also built on the workers' mobility network. In this paper, I explicitly connect the firms' centrality in the job-to-job network to their connectivity parameter in the model, thereby investigating a novel channel that firms use to deliver value to workers.

This paper also draws on several studies that address the importance of labor market frictions through random search models with sequential auctions. Specifically, the bargaining protocol comes from the seminal works of Postel-Vinay and Robin (2002) and Cahuc et al. (2006). Similar to Gregory (2023) and Jarosch (2023), firms in my model are heterogeneous along a second dimension other than productivity (in their cases, the quality of the learning environment and job security, respectively). In my model, the mechanism through which workers value firms' connectivity pertains to the reduction in search frictions, which provides workers with a higher likelihood of better opportunities, independent of the human capital dynamic.

Lastly, mechanisms closely related to the economic intuition behind the role of firms' connectivity, such as screening and sorting, are investigated by Cai et al. (2021) in a search model with information frictions. However, they focus on the strategic decision of the firm regarding the optimal size of screening pools rather than assessing how employers' screening capacities may explain wage heterogeneity.

Outline. Section 2 introduces a random search model with firm heterogeneity in both productivity and connectivity. Section 3 outlines the data sources, constructs a measure for firm-specific outside offers, and tests the model's key reduced-form predictions. Section 4 discusses identification and estimation. Section 5 examines the efficiency implications of the model's implied job ladder, while Section 6 performs counterfactual exercises by shutting down the model's core mechanism. Section 7 concludes.

2 — A Search model with connectivity

This section introduces an equilibrium model of the labor market that accounts for heterogeneity among both workers and firms. Firms are uniquely characterized by two attributes: productivity and connectivity. This model's novelty lies in the inclusion of the latter, which governs the firm-specific likelihood of employed workers receiving or making job offers during their on-the-job search.

2.1 – Heterogeneous agents

The market consists of a continuum of workers who are infinitely-lived and differentiated by ability, denoted a. These abilities are distributed exogenously over a continuous set $[\underline{a}, \bar{a}]$, following a cumulative distribution function $A(\cdot)$. Workers have linear preferences for a single good and can either be employed or unemployed.

On the other side of the market, firms are represented by the type $\boldsymbol{\theta} = (\theta_p, \theta_c)$, where θ_p and θ_c represent the firm's productivity and connectivity, respectively. These parameters are distributed exogenously according to cumulative distribution functions $P(\cdot)$ over $[\underline{\theta}_p, \bar{\theta}_p]$ and $T(\cdot)$ over $[\underline{\theta}_c, \bar{\theta}_c]$, respectively. Their joint distribution is denoted as $F(\cdot)$. Workers and firms alike discount future returns at a common rate β.

2.2 – Meetings and production

Time is discrete. Both workers and firms search on the market for (possibly better) matches, while unemployed workers search for employment. This search process is random, undirected, and incurs costs. Workers and firms meet at each period.

Employed worker. In conventional random search models, workers seeking job opportunities receive external offers with a consistent probability that is independent of the types of firms and might be linked to workers' characteristics through an individual search effort. When firms are indistinguishable from jobs, one can interpret such a meeting mechanism as a firm-to-firm interaction: the current and potential employer meet, and the worker observes the resulting offer with an exogenous probability. In the model, a meeting between two firms does not guarantee an employed worker a job offer. The formalization of the offer depends on the connectivity of both the current employer (the "incumbent") and the potential employer (the "challenger"). For the worker to be aware of the interaction between the firms—and therefore the offer—the combined connectivity of both firms must exceed an exogenous threshold, denoted as I. Consequently, a worker employed at firm B will receive a valid job offer from firm A only if the combined connectivity of firms A and B is larger than I: $\theta_c^A + \theta_c^B \geq I$.

This simple model's characteristic attempts to formalize a widely observed labor market phenomenon: the rate at which workers receive job offers varies depending on their current employment. The higher the connectivity of the incumbent firm, the higher the likelihood of the worker receiving an external offer. Similarly, firms with extensive connectivity are more likely to engage workers employed at incumbent firms. The economic rationale behind this process can be interpreted in several ways. For instance, firms with higher connectivity may systematically provide higher-quality workers due to superior rates of firm-specific human capital accumulation or more efficient screening technologies. Alternatively, companies with extensive connectivity may reduce search frictions for potential future employers because of existing business relationships, such as sales and purchases. Variations in connectivity could also reflect differences in corporate culture. For example, some companies actively assist their former employees in securing new employment. For now, I will abstract from discussing one specific underlying mechanism that can explain why job offer rates vary across employers.

Unemployed worker. In this model, unemployment is defined as a firm characterized by a productivity-connectivity pair $u = (u_p, 0)$, where $u_p < \theta_p$ for any given p. The connectivity parameter is irrelevant in the context of unemployment, which justifies setting it to zero. As a result, the rate at which an unemployed worker receives job offers, denoted as λ, is independent of the firm's characteristics in attempting to hire the worker. This rate is considered exogenous, highlighting that connectivity does not influence the job offer rate for unemployed individuals, as in a standard random search model.

2.3 – Wage setting protocol

In this model, wages are viewed as fixed contracts that can be renegotiated under specific conditions, particularly when credible threats arise. These threats might occur when workers receive an external job offer substantial enough to be leveraged for renegotiating their current wage with their existing employer or when they transition to a new company. When such a formal offer is made, the incumbent and challenging firms engage in Bertrand competition for the worker, making repeated bids. This sequential auction mechanism was initially proposed by Postel-Vinay and Robin (2002) and later refined by Cahuc et al. (2006). The notation used in what follows is partly borrowed from Jarosch (2023).

Let W, U, and J denote the value of an employed worker, an unemployed worker, and a job for a firm, respectively. $S(a, \theta_p, \theta_c)$ represents the joint surplus generated by a match between a worker of type a and a firm of type (θ_p, θ_c). As in the classic sequential auction setting, both the worker's wage $w\left(a, \theta_p, \theta_c, \hat{\theta}_p, \hat{\theta}_c\right)$ and the value $W\left(a, \theta_p, \theta_c, \hat{\theta}_p, \hat{\theta}_c\right)$ depend on the worker's ability a, the current employer's type (θ_p, θ_c), and the type of the firm involved in the last wage negotiation, $(\hat{\theta}_p, \hat{\theta}_c)$.

Unemployed worker. If an unemployed worker forms a match with a firm $\boldsymbol{\theta} = (\theta_p, \theta_c)$, the wage should satisfy

$$W(a, \theta_p, \theta_c, u_p, 0) - U = \sigma S(a, \theta_p, \theta_c) \tag{1}$$

where $\sigma \in [0,1]$ represents the worker's bargaining power over the match surplus. As $S(a, u_p, 0) = 0$, the set of firms an unemployed worker is willing to work for is represented by $\mathcal{F}_1(u) \equiv (\theta_p, \theta_c) \mid S(a, \theta_p, \theta_c) > 0$.

Employed worker. For an employed worker of type a currently working at the incumbent firm $\boldsymbol{\theta}^1 = (\theta_p^1, \theta_c^1)$, three mutually exclusive cases may arise if challenged by a firm $\boldsymbol{\theta}^2 = (\theta_p^2, \theta_c^2)$.

1. The worker produces a higher joint surplus with the firm (θ_p^2, θ_c^2) than with the firm (θ_p^1, θ_c^1), i.e., $S\big(a, \theta_p^2, \theta_c^2\big) > S\big(a, \theta_p^1, \theta_c^1\big)$. As a result, the incumbent employer becomes the new negotiation benchmark, and the worker transitions to the challenger firm (θ_p^2, θ_c^2) with a wage such that[3]

$$W\big(a, \theta_p^2, \theta_c^2, \theta_p^1, \theta_c^1\big) - U = S\big(a, \theta_p^1, \theta_c^1\big) + \sigma\left[S\big(a, \theta_p^2, \theta_c^2\big) - S\big(a, \theta_p^1, \theta_c^1\big)\right] \tag{2}$$

   The worker, therefore, receives the whole surplus of the incumbent match plus a share σ of the net gains from the movement to θ_2. For a worker employed at θ, the set of firms that allow this first case is $\mathcal{F}_1(\theta_p, \theta_c) \equiv \big\{(\theta_p', \theta_c') \mid S\big(a, \theta_p', \theta_c'\big) > S(a, \theta_p, \theta_c)\big\}$. This set thus includes all firms where the surplus generated together with the worker is greater than the surplus generated at the incumbent firm.

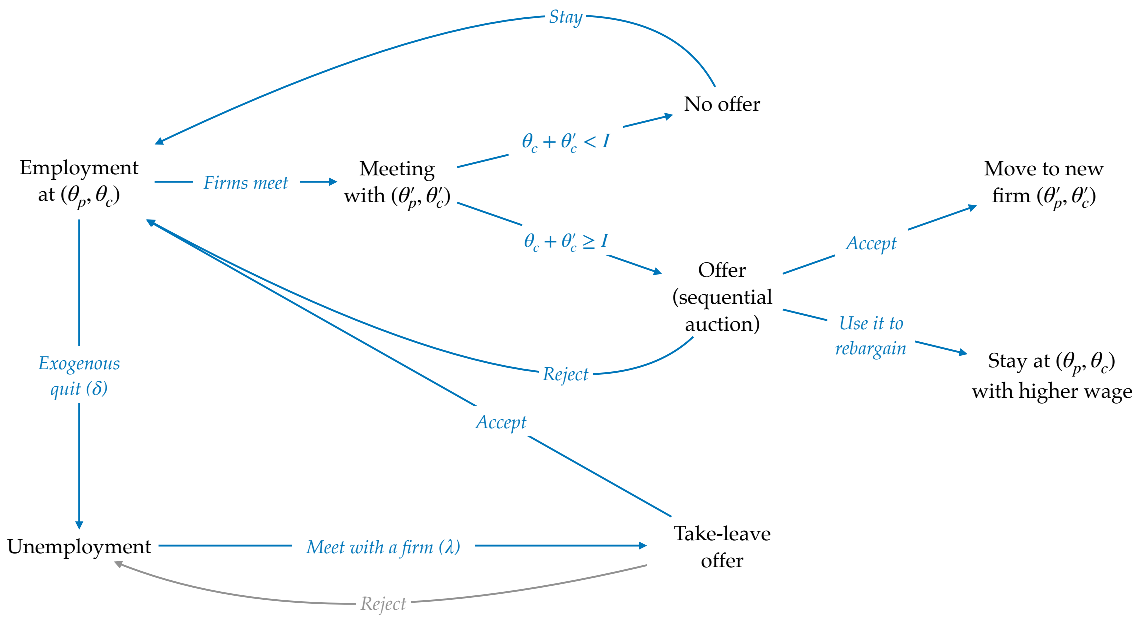
2. The worker produces a higher joint surplus with the incumbent after a renegotiation. This would occur if $S\big(a, \theta_p^2, \theta_c^2\big) < S\big(a, \theta_p^1, \theta_c^1\big)$, but the current negotiation benchmark is still lower than $S\big(a, \theta_p^2, \theta_c^2\big)$. This means the challenging firm could offer a wage that's more attractive than the worker's current wage. The worker could then use this external job offer to negotiate a higher salary while staying with the incumbent firm. The incumbent firm would then have to raise the worker's salary just enough to keep them. The new wage would meet the following indifference condition:

$$W\big(a, \theta_p^1, \theta_c^1, \theta_p^2, \theta_c^2\big) - U = S\big(a, \theta_p^2, \theta_c^2\big) + \sigma\left[S\big(a, \theta_p^1, \theta_c^1\big) - S\big(a, \theta_p^2, \theta_c^2\big)\right] \tag{3}$$

   I will refer to the set of firms where this second scenario applies as $\mathcal{F}_2\Big(\theta_p, \theta_c, \hat{\theta}_p, \hat{\theta}_c\Big) \equiv \Big\{(\theta_p', \theta_c') \mid S(a, \theta_p, \theta_c) > S\big(a, \theta_p', \theta_c'\big) > S\Big(a, \hat{\theta}_p, \hat{\theta}_c\Big)\Big\}$.

3. As I will extensively discuss later on, the new wage a worker obtains upon a movement can be *lower* than the wages set with the incumbent due to a compensating differential mechanism.

Figure 1 — Diagram of the model



Notes. A firm is a productivity-connectivity couple (θ_p, θ_c). Employed workers receive offers if the connectivities of the incumbent and challenging firm are sufficiently large. Unemployed workers receive offers with exogenous probability λ and do not reject job offers. Matches are destroyed at the exogenous rate δ. Employed workers who receive an offer decide whether to move to the challenging firm, use the outside offer to re-bargain their wage with the incumbent firm or discard it, as explained in Section 2.3.

3. The value generated by the offer is entirely dominated by the current negotiation benchmark, i.e., the previous outside option. In this case, the surplus a worker could generate with the challenging firm is less than what they could generate with the incumbent firm. Moreover, the worker cannot use the external job offer to negotiate a higher wage. As a result, the worker simply dismisses the offer and continues to work for the incumbent firm at the same wage.

The sequential auction wage setting protocol outlined above generates frictional wage dispersion and governs both wage dynamics and job-to-job transitions, depending on the worker's recent employment history (their negotiation benchmark). As long as workers remain employed, they ascend the job ladder by transitioning to firms that offer increasing value. They also utilize external job offers to influence their wage dynamics, taking advantage of these opportunities to negotiate higher wages and secure better positions.

Figure 1 provides a graphical representation of the model's dynamics.

2.4 – Value functions

This paragraph illustrates the value functions that summarize the model previously outlined. To enhance readability, in what follows $\boldsymbol{\theta} = (\theta_p, \theta_c)$ and $\hat{\boldsymbol{\theta}} \equiv (\hat{\theta}_p, \hat{\theta}_c)$. This approach, while constituting a minor abuse of notation—given that the value functions' arguments change in number depending on context—leads to a cleaner expression of equations.

Employed worker. The value of being employed for a worker of ability a at a firm $\boldsymbol{\theta}$ with a negotiation benchmark $\hat{\boldsymbol{\theta}}$ can be expressed by The employment value for a worker of ability a at a firm $\boldsymbol{\theta}$ with negotiation benchmark $\hat{\boldsymbol{\theta}}$ is

$$\begin{aligned} W\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) &= w\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) \\ &+ \beta \left\{ (1-\delta) \left[\int_{I-\theta_c}^{\bar{\theta}_c} \left(\int_{x \in \mathcal{F}_1(\theta_p, y)} W(a, x, y, \boldsymbol{\theta}) \, \mathrm{d}P(x) \right.\right.\right. \\ &+ \left. \int_{x \in \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} W(a, \boldsymbol{\theta}, x, y) \, \mathrm{d}P(x) \right) \mathrm{d}T(y) \\ &+ \left.\left.\left(1 - \int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_1(\theta_p, y) \cup \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) W\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) \right] + \delta U \right\} \end{aligned} \tag{4}$$

The interpretation of equation (4) follows the wage-setting protocol. The value of employment is comprised of the current wage $w\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right)$, plus a discounted future value that takes into account the possibility of exogenous job loss, which happens with a probability of δ. If the worker maintains their employment and the combination of incumbent and challenger firms meets the necessary connectivity threshold, three mutually exclusive outcomes can occur: *a)* The worker may receive an offer from a firm in set $\mathcal{F}_1(\boldsymbol{\theta})$, thus opting to move to this new firm and establishing the incumbent firm as the new negotiation benchmark; *b)* The worker may receive an offer from a firm in $\mathcal{F}_2\left(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right)$, allowing them to stay with their current employer but with an updated negotiation benchmark and wage; *c)* The worker may choose to remain in their current position with no changes to the negotiation benchmark or wage. If employment ends, the worker transitions to unemployment and receives a flow of income of au_p, which they must relinquish upon gaining new employment.

Unemployed worker. The value for an unemployed worker is described by

$$
\begin{aligned}
U = u + \beta \Bigg[\lambda \iint_{x,y \in \mathcal{F}_1(u)} & W(a, x, y, u_p, 0) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \\
+ \left(1 - \lambda \iint_{x,y \in \mathcal{F}_1(\boldsymbol{\theta})} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) U \Bigg]
\end{aligned}
\tag{5}
$$

Unemployed workers receive an income flow of u, irrespective of their ability. With a probability of λ, they may receive an offer that they will invariably accept. If no offers arrive, their continuation value remains the same as the current value of unemployment. It is crucial to note that the connectivity mechanism does not apply to unemployed workers. This is due to the assumption that unemployment does not carry any connectivity attributes. Furthermore, unemployed workers will accept any job offer they receive, regardless of the offering firm's connectivity or productivity.

Firm. The value for a firm $\boldsymbol{\theta}$ matched with a worker a who has a negotiation benchmark $\hat{\boldsymbol{\theta}}$ is given by

$$
\begin{aligned}
J\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) = {} & f(a, \theta_p) - w\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) \\
& + \beta \left(1 - \delta\right) \Bigg[\int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} J(a, \boldsymbol{\theta}, x, y) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \\
& + \left(1 - \int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_1(\theta_p, y) \cup \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) J\left(a, \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right) \Bigg]
\end{aligned}
\tag{6}
$$

The value for the firm includes its current profit (the match's production less the wage) and the continuation value of employing the worker. Should workers receive an offer from a challenging firm selected from $\mathcal{F}_2\left(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}\right)$, they stay with the incumbent employer but with updated wages. Since matches cease with worker departure, the firm does not receive any future value once the worker leaves, which happens both in the case of exogenous separation and the worker moving to a better firm. If no offers are presented or the offer is rejected, the match remains unchanged, and the continuation value for the subsequent period is simply the discounted current value.

Joint surplus. Assuming free entry, the joint surplus generated by a worker with ability a, matched with a firm $\boldsymbol{\theta}$, can be defined as the sum of the worker's and firm's values, minus the unemployment value. Combining the three Bellman equations and applying the bargaining protocol enables us to express it as follows:

$$
\begin{aligned}
S(a,\boldsymbol{\theta}) &= \max\left\{0, W\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right) - U + J\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right)\right\} \\
&= \max\left\{0, f(a,\theta_p) - u + \beta\left[(1-\delta)\left(S(a,\boldsymbol{\theta})\right.\right.\right. \\
&\left. + \sigma\int_{I-\theta_c}^{\bar{\theta}_c}\int_{x\in\mathcal{F}_1(\theta_p,y)}\left[S(a,x,y)-S(a,\boldsymbol{\theta})\right]\,\mathrm{d}P(x)\,\mathrm{d}T(y)\right) \\
&\left.\left. - \sigma\lambda\iint_{x,y\in\mathcal{F}_1(u)} S\left(a,x,y\right)\,\mathrm{d}P(x)\,\mathrm{d}T(y)\right]\right\}
\end{aligned}
\tag{7}
$$

The continuation value of the joint surplus accounts for the option value of on-the-job search, which can be delivered through both dimensions of the firm. The continuation value is the sum of the present value of the joint surplus and an additional term, which accounts for the fact that a worker transitioning to another firm not only receives the total surplus of the current match but also a fraction, denoted by σ, of the net surplus gains. It is important to note that since the present component of the value function is already net of the unemployment benefit that would be forfeited, the future value is likewise net of the optional value of search during unemployment that would be foregone. Furthermore, the surplus is independent of the negotiation benchmark $\hat{\boldsymbol{\theta}}$. This is because, under transferable utility, the distribution of the rents within the match does not change its value. Hence, wages, being a pure within-match redistribution, do not enter the equation. Finally, the surplus is strictly increasing in both θ_p and θ_c, ranking jobs across productivity and connectivity according to their appeal to workers.

Equation (7) governs all worker transitions, including between employment and unemployment, as well as between different firms. Crucially, these transitions are independent of the distribution of workers across different states, which considerably reduces computational effort when numerically solving the equation.

2.5 – Wage equation

It is possible to solve the model to derive a convenient closed-form wage equation. This equation pins down the wages showing how they deliver values according to the wage-setting protocol, as delineated in Section 2.3 for each incumbent-negotiation benchmark firm pair. The wage equation, along with the surplus value function (7), oversees the earnings dynamics for each worker's labor market history, just as the surplus value function regulates worker flows.

I build the wage equation exploiting the wage setting protocol given by equations (1)-(3) together with the surplus value function given by (7) and the employed (4) and unemployed (5) value functions, as detailed in Appendix E. The equation reads as

$$
\begin{aligned}
w\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right) = \kappa + \sigma f(a,\theta_p) - \beta\left(1-\delta\right)\Bigg(& \underbrace{G\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right)}_{\text{Gains from otj search}} \\
& - \underbrace{\sigma^2 \int_{I-\theta_c}^{\bar{\theta}_c} \int_{x\in\mathcal{F}_1(\theta_p,y)} \left[S(a,x,y) - S(a,\boldsymbol{\theta})\right] \mathrm{d}P(x)\,\mathrm{d}T(y)}_{\text{Gains from new employer}}\Bigg) \\
& + \underbrace{\left[1-\beta\left(1-\delta\right)\right]\left(1-\sigma\right) S\left(a,\hat{\theta}\right)}_{\text{Current benchmark surplus}}
\end{aligned} \tag{8}
$$

The wage equation is composed of four main terms. First, a fixed component k that gathers the terms that do not depend on $\boldsymbol{\theta}$ and mostly accounts for unemployment foregone value. Second, a present value of the match, i.e., a share σ of the flow of output produced. Third, a continuation term that accounts for all the future value provided by the firm. In its turn, it is composed of three terms: the pure new-employment gains—increasing the present value of the wage—, the gains from having the current benchmark value while searching, and the value related to the possibility of searching on the job from the current position. The last two terms reduce the present value of the wage by discounting their future contribution to the worker's salary. More in detail, the function $G\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right)$ encapsulates the worker's gains from on-the-job search:

$$
\begin{aligned}
G\left(a,\boldsymbol{\theta},\hat{\boldsymbol{\theta}}\right) = \int_{I-\theta_c}^{\bar{\theta}_c} \Bigg(& \int_{x\in\mathcal{F}_2\left(\theta_p,y,\hat{\boldsymbol{\theta}}\right)} \overbrace{\left(1-\sigma\right)\left[S(a,x,y) - S\left(a,\hat{\boldsymbol{\theta}}\right)\right]}^{\text{Re-bargaining with the incumbent}} \mathrm{d}P(x) \\
& + \int_{x\in\mathcal{F}_1(\theta_p,y)} \underbrace{\left(1-\sigma\right)\left[S(a,\boldsymbol{\theta}) - S\left(a,\hat{\boldsymbol{\theta}}\right)\right]}_{\text{New negotiation benchmark}} \\
& + \underbrace{\sigma\left[S(a,x,y) - S(a,\boldsymbol{\theta})\right]}_{\text{New employer}} \mathrm{d}P(x)\Bigg) \mathrm{d}T(y)
\end{aligned}
$$

The on-the-job search component delivers value to the worker through three distinct channels. First, employees can leverage viable outside options to renegotiate the current wage with the incumbent firm. Second, workers who choose to transition to the challenging firm establish a new negotiation benchmark, thereby setting a precedent for the incumbent firm. Finally, the transitioning workers gain rents from the difference between the surplus generated with the new firm and with the incumbent.

The on-the-job-search component reduces the wages as per equation (8), as the prospective value of searching from the firm is discounted upon transition. A similar reasoning applies when a new negotiation benchmark is established due to an outside offer from a competing firm. Furthermore, wages exhibit an inverse correlation with the firm's connectivity θ_c. This is attributed to the surplus splitting mechanism that creates compensating differentials. In essence, workers are willing to accept lower present wages in exchange for potential future opportunities arising from increased meeting probabilities, leading to quicker advancement on the job ladder either through re-negotiations utilizing outside options or through job-to-job transitions. This purely-compensating differentials effect echoes the one of Jarosch (2023), where workers trade-off wages for job security, and the one of Gregory (2023), where workers are compensated through faster rates of human capital accumulation, given their age. Still, the mechanism through which the worker improves its future value is entirely different in my model, as it entirely attains the heterogeneity in the firm-specific offers' arrival rates.

On the other hand, the relationship between a firm's productivity θ_p and wages is ambiguous and dependent on the worker's bargaining power σ. Indeed, θ_p influences wages in two significant ways: directly, where more productive firms command higher wages due to increased output; and indirectly, with more productive firms promising greater future wage growth—a compensating differential mechanism similar to the one associated with connectivity. Consider the two extreme scenarios for clarity. In the instance where workers possess no bargaining power ($\sigma = 0$), the hiring wage is set to compensate for the entire surplus from their previous employment upon transitioning to a new firm. Consequently, as the new firm's productivity type increases, there is a larger scope for future wage growth through on-the-job search gains, which, in turn, lowers the current wage. Essentially, the firm is discounting the future wage growth it offers to the worker. Contrarily, if $\sigma = 1$, indicating that workers have the entire bargaining power, workers get the whole surplus, the on-the-job gains only become significant upon transitions, and the value delivered through wages matches the employer's productivity. This signifies that more productive firms yield higher wages. This uncertain relationship between productivity and wages is a well-established outcome of sequential auctions random search models, as first presented in Cahuc et al. (2006).

2.6 – Equilibrium

Given the exogenous distributions $A(a)$, $P(\theta_p)$ and $T(\theta_c)$, a steady-state equilibrium is:

- a surplus function $S(a, \theta_p, \theta_c)$ satisfying the Bellman equation given in (7);
- a worker net surplus function $W\left(a, \theta_p, \theta_c, \hat{\theta}_p, \hat{\theta}_c\right) - U$ satisfying the bargaining protocol given by equations (1), (2) and (3);
- a wage equation $w\left(a, \theta_p, \theta_c, \hat{\theta}_p, \hat{\theta}_c\right)$ satisfying (E1);
- a steady state distribution of workers across employment states such that
  - inflows of workers equate outflows of workers
  - the distribution of workers across employment and unemployment states evolves according to the wage-setting rules and the transitions determined by the surplus value function.

I borrow the convenient notation from Jarosch (2023) in calling $g\left(\theta, \hat{\theta}\right)$ the density of workers employed in a firm θ with negotiation benchmark $\hat{\theta}$, $g(\theta, u)$ the density of workers in a firm θ with benchmark unemployment, and u the measure of unemployed workers. Then, in equilibrium, one has the following set of flow balances:

$$
\begin{aligned}
g^{-}\left(\theta, \hat{\theta}\right) &= g^{-}\left(\theta, \hat{\theta}\right)\left(\delta + (1-\delta)\int_{I-\theta_c}^{\bar{\theta}_c}\int_{x\in\mathcal{F}_1(\theta_p, y)\cup\mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} \mathrm{d}P(x)\,\mathrm{d}T(y)\right) \\
g^{+}\left(\theta, \hat{\theta}\right) &= f(\theta)\left[\mathbb{1}_{\theta\in\mathcal{F}_1(\hat{\theta})}(1-\delta)\left(\int g(\hat{\theta}, x)\,\mathrm{d}x + g\left(\hat{\theta}, u\right)\right)\right] \\
&\quad + f\left(\hat{\theta}\right)\left[\int \mathbb{1}_{\hat{\theta}\in\mathcal{F}_2(\theta, x)}(1-\delta) g(\theta, x)\,\mathrm{d}x\right] \\
g^{+}(\theta, u) &= \lambda u f(\theta) \\
g^{-}(\theta, u) &= g(\theta, u)\left(\delta + (1-\delta)\left[\int_{I-\theta_c}^{\bar{\theta}_c}\left(\int_{x\in\mathcal{F}_1(\theta_p, y)} \mathrm{d}P(x) + \int_{x\in\mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} \mathrm{d}P(x)\right)\mathrm{d}T(y)\right]\right) \\
u^{+} &= \delta \iint g(x, y)\,\mathrm{d}x\,\mathrm{d}y \\
u^{-} &= \lambda\left(1 - u\right)
\end{aligned}
$$

where $\mathbb{1}$ is the indicator function.

2.7 – Model discussion

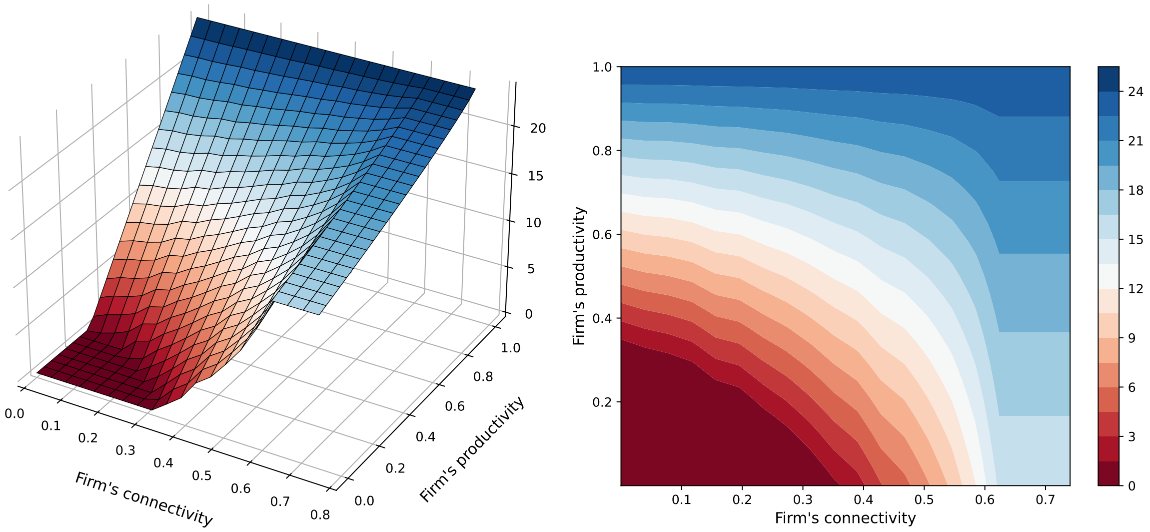
I next discuss some relevant properties of the model.

Productivity/connectivity trade-off. Workers receive value from the firm along its two heterogeneity dimensions. As equation (7) shows in its first term, the firm's productivity in a match directly increases the surplus generated, given the worker's ability. The higher the firm's productivity, the higher the output yielded and, therefore, the higher the value for the worker. Conversely, a higher firm's connectivity does not translate into any present worth for the workers; it only increases their future likelihood of receiving more offers. It follows that a worker values a firm's connectivity as long as it cannot convey direct value through its productivity. Thus, a less productive firm can still attract employees thanks to its connectivity, reducing their on-the-job search frictions, thereby increasing their likelihood of meeting a higher-productivity firm later on. Matching with a high-connectivity firm essentially allows the workers to climb the job ladder faster, improving their probability of meeting a 'good firm' sooner.

Figure 2 shows this productivity-connectivity trade-off for three different levels of the meeting threshold I. It plots indifference curves for a worker of a given ability as a function of the two firm's attributes. The Figure shows that a worker demands higher compensation in terms of connectivity for less productive firms. Notably, workers disregard the connectivity of productive-enough firms since they are satisfied with staying in an establishment that directly delivers considerable value. This property has intriguing implications for a worker's life cycle, even if the model does not account for it straight away. Indeed, workers tend to sort into higher-productive firms as they climb the job ladder, implying that they attach a higher value to a firm's connectivity in the earlier stage of their career.

Figure 2 also shows that the relative importance of a firm's connectivity depends on how easy it is to concrete the meeting between the incumbent and challenging firms, i.e., how easy it is to formalize the offer. The third panel of the Figure gives the intuition behind this result, displaying that the connectivity compensation for low-productivity firms is higher when it is more challenging to observe successful meetings since workers discount that low-connectivity firms will not likely impact future opportunities. Therefore, they are willing to forego more connectivity for productivity than in cases where the threshold is lower.

Figure 2 — The productivity/connectivity trade-off



(a) 3D plot **(b) Iso-values**

Notes. The figure displays a 3D plot (panel a) alongside indifference curves (panel b) to depict the joint surplus as a function of a firm's connectivity (x-axis) and productivity (y-axis) for a specific worker type. This illustrates the trade-off between a firm's productivity and its connectivity. Workers value a firm's connectivity unless the firm's productivity is sufficiently high to offset the need for it. As a result, less productive firms can attract workers by offering higher connectivity, which reduces search frictions and expedites transitions to more productive firms. On the other hand, when workers find highly productive firms, they assign less importance to connectivity due to the already high wages. Hence, the significance of the connectivity channel shifts depending on the connectivity threshold. These plots are generated by numerically solving equation (7) on a 25x25 grid for a specific worker ability draw. The marginal distributions of the firms' attributes are as estimated and presented in Table 1. Details on the estimation procedure can be found in Section 4.

Endogenous mobility and the job-to-job network. The meeting mechanism between firms described in Section 2.2 implies that the higher the connectivity of a firm, the higher its degree centrality in the endogenous job-to-job network generated by the model after sufficient iterations.[4] This intuition is formalized as follows.

Proposition 1. The connectivity parameter c maps into the degree centrality of the network $G = (V, E)$ where vertexes V are firms and edges E are workers' transitions across firms.

Proof. The proof can be found in Appendix E. $\square$

Due to the mobility dynamics implied by the meeting mechanism, firms with higher connectivity parameters will eventually trade more workers than firms with lower connectivity—

4. Degree centrality, in graph theory, is a measure of a node's importance based on the number of edges it has, i.e., it is the number of its direct connections. I will extensively discuss the concept of degree centrality and how I use it in the context of connectivity in Section 3.2.2.

both in hiring and relinquishing. Moreover, firms with higher connectivity will exchange workers more frequently with other similar firms. Thus, after enough iterations, the higher the θ_c parameter, the higher the degree centrality value in the job-to-job network. This is true for both out-degree and in-degree measures. I exploit this close relationship to build reduced-form results that align with predictions of the model, and to inform the parameters related to the connectivity's distribution when estimating the model, as detailed in Section 4.1.

Sorting. The model has no predicted sorting since the production function is additively separable in my setup. Indeed, there are no complementarities between the workers' ability and the firms' productivity. Given two firm types θ and θ', it never happens that θ is preferred to θ' for some workers and the other way around for others. All matches generate a positive surplus, and there exists a wage always acceptable for every worker-firm couple.

3 — Empirical evidence of the model's main mechanisms

This section provides reduced-form evidence of some of the model's implications, which leverage the link between the connectivity parameter of the model and the firm's degree in the job-to-job network. It is composed of three parts. First, it presents the data sources used to empirically verify the model's predictions. Second, it outlines the construction of the job-to-job network and explains how it can be used to identify firms that play a relatively more significant role in employee transitions in terms of degree centrality. Third, it provides and discusses evidence of three main predictions of the model regarding the role of the connectivity channel in the labor market.

3.1 – Matched employer-employee data

This paper relies on confidential administrative datasets provided by the Italian National Social Security Institute (Istituto Nazionale di Previdenza Sociale, INPS). More specifically, it draws on a comprehensive, matched employer-employee dataset comprising monthly-level data for all non-agricultural private firms in Italy that employ at least one salaried worker. Each worker-firm record provides detailed insight into various aspects of the employment match, including contract start and end dates, reasons for commencement or termination, contract type, work schedule, employee's occupational category, earnings, and actual days worked. This employer-employee dataset is supplemented with additional detailed information at both the worker level—like demographic characteristics—and the firm level—such as industry, location, and key dates of the firms' lifespan.

The analysis is restricted to active contracts from 2008 to 2020, specifically those in firms that employed no fewer than 15 workers at least once during this period. I focus on large firms, arguing they can better convey connectivity value compared to smaller firms, which often lack the necessary organizational infrastructure for significant connectivity amenities.[5] My sample includes full-time contracts among employees who held a white-collar position within a sample firm for a minimum of one year during the period under analysis. The goal is to focus on employees who play pivotal roles in their firms' operations and stand to gain the most from connectivity channels. The monthly earnings of an employee are unaffected by transitory shocks such as leaves of absence and bonuses.

Appendix B details and discusses the data cleaning decisions. This process ultimately yields a quarterly panel of 2,742,853 workers across 197,347 firms between 2008 and 2020.

3.2 – The job-to-job network

I exploit the panel structure of the dataset to obtain detailed information about worker movements between firms throughout the sample period. By following workers' movements across firms over time, I can reconstruct the job-to-job transition network from the panel between 2008 and 2018. In this network, firms are nodes and directed links between nodes represent the movements of workers between firms. Thanks to this structure, I will compute the degree centrality of each firm in the sample period, thus exploiting the result obtained in Proposition 1 to link a primitive of the model to a measurable labor market quantity.

More formally, the job-to-job network $N = \{V, E\}$ consists of the set of nodes V—the firms in the sample involved in at least one job-to-job transition in the reference period—and a set of links E—the workers' movements between firms. An adjacency matrix A can represent this network, where $A_{ij} = 1$ if at least one worker moves from firm i to firm j.[6] I define a movement as a worker changing between two firms within no more than two months from quitting the old firm and starting at the new one. The granular information on the reason

5. More precisely, I further restrict the sample to workers who worked exclusively in large firms during the reference period.

6. I abstract from the network weights, i.e., the strength of links based on the number of workers flowing from one firm to another. I will relax this when considering centrality measures that account for weights, considering an adjacency matrix A such that $A_{ij} = k$ where $k > 0$ is the number of workers flowing from i to j in the reference period.

behind a spell's start or end allows me to identify proper job-to-job movements, distinguishing them from layoffs or changes in the firm's identifier due to internal reorganization.[7] The job-to-job network is a *directed* network since the connections between its nodes are directional and, in general, $A_{ij} \neq A_{ji}$ for every i and j.

3.2.1 Descriptive statistics: panel and network

Descriptive statistics of the job-to-job transitions are detailed in Table A1 for the entire period, with further breakdown into four-year sub-periods. The demographic composition of workers transitioning between employers remains consistent over time, with a notably low proportion of women involved in movements (Rubolino, 2022). Both average tenure and age at the time of a standard movement increased over time, whereas the age at the first movement decreased by nearly 1.5 years from the 2008-2011 to 2017-2020 periods. Workers showed a decreasing trend in transitioning within the same industry and province, indicative of broadening labor markets. Interestingly, minor average wage cuts (around 3% across the entire period) are associated with movements, possibly reflecting changes in the non-monetary dimension of job value (Caplin et al., 2022). In total, nearly 1.5 million workers transitioned across at least two firms between 2008 and 2020.

Table A2 provides descriptive statistics for the network, once again segmented into four-year sub-periods. The number of between-firm movements declined over time, mirroring the decrease in nodes and links within the sub-networks. The table also outlines the number of connected components in the network.[8] Additionally, restricting to this sub-network incurs a small sampling cost, as the largest connected component comprises 91.1% of nodes and 99.2% of links.

Table A3 presents descriptive statistics of the sample in the entire panel and within the largest connected component. Firms in the latter are, on average, larger but younger. The largest component has a higher share of firms operating in the services sector than the full panel; however, more than two-thirds of employers are still in manufacturing. Demographic characteristics, as well as average tenure and experience, remain consistent across both pan-

7. For example, a firm changing its business name or tax code for fiscal reasons changes its identifier in the administrative data, potentially leading to an *apparent* job-to-job transition, even though the worker remains in the same firm. Given the information I have on the motivation behind a movement, I avoid this risk.

8. A connected component consists of a network's subset of nodes, such that a path connects each pair. In this paper, my focus is on the largest connected set or the connected component containing the most nodes. Such a restriction is meant to focus on the most significant part of the network, where the centrality analysis I propose is most relevant.

els. Workers in the largest connected set earn marginally more, reflecting the presence of larger firms. Overall, the descriptive evidence demonstrates significant consistency between the entire panel and the largest connected component in the job-to-job network.

3.2.2 Firms' per-worker degree centrality in the network

I employ the job-to-job network to examine the extent to which firms intermediate worker flows across a variety of sources and destinations. Proposition 1 links a firm's connectivity parameter in the model to the degree centrality of the job-to-job network. Degree centrality (Freeman, 1978) is the number of other nodes each node connects to in the directed network. Formally, for a node i in a network with total nodes N, it reads as $D(i) = \sum_j^N A_{ij}$ where A is the adjacency matrix and $A_{ij} = 1$ if a link between i and j exists. Conceptually, a firm's relative significance depends on its capacity to link workers with other firms—thus effectively "controlling" the flows between employers.

Degree centrality offers a simple measure of a node's participation in a network, as it relies solely on the local structure surrounding it. In the case of a directed network like the job-to-job one, it is natural to divide degree centrality into in-degree and out-degree—in this context, the number of in- and out-going links a firm has.[9] To further clarify, a firm that sends workers to 10 different firms over the sample period will have an out-degree centrality of 10. Likewise, a firm that receives workers from 8 different firms within the same period will have an in-degree centrality of 8. In the case of an unweighted network—one that does not account for the strength of links between nodes—degree centrality only considers the variety of connections each node possesses, disregarding the *intensity* of worker flows.

The empirical analysis in this paper will primarily focus on out-degree centrality, as it best conceptually aligns with the model's connectivity parameter. I interpret out-degree centrality as an empirical measure of the variety of job offers received by employees of a given firm over time. Still, taking into account in-degree centrality helps distinguish firms that are frequently left for a multitude of other destinations due to their low quality (i.e., workers might end up there due to labor market frictions and wish to leave as soon as possible) from those that workers deliberately choose. However, larger firms often have, on average, a greater degree centrality for mechanical reasons unrelated to the economic intuition behind what I

9. As discussed by Borgatti (2005), degree centrality measures are particularly suited for walk-based transfer processes along the graph, which applies to job-to-job networks.

have termed 'connectivity': in this paper, I am not examining the relationship between an employer's size and worker wages. Thus, I re-scale each firm's centrality by its average number of employees during the sample period:

$$D^{\text{pw}}(i) = \frac{1}{\bar{n}} \sum_{j}^{N} A_{ij} \quad A_{ij} = 1 \text{ if } i \text{ is linked to } j$$

This adjustment ensures that a firm is considered more central in the network if it truly maintains richer connections with other firms, rather than just being larger. Moreover, such a normalization allows for an interpretation of the per-worker centrality measure as a proxy for *individual* job opportunities employees have received over time in a given firm.

Table A4 presents summary statistics of the worker-firm panel, categorized by quartiles of normalized out-degree centrality. On average, firms with high centrality employ younger workers, initiate contracts earlier, and offer higher salaries compared to firms at the lower end of the out-degree centrality distribution. Moreover, firms with greater centrality tend to hire more foreign workers and fewer female workers. Notably, firm-specific tenure decreases with centrality, potentially suggesting that central firms serve as "springboards" for workers' future career trajectories. In addition, Figure A2 displays the mean values of relevant financial measures from the Cerved database, categorized by the ventile of both out- and in-degree centrality. Firms with higher centrality show lower levels of tangible assets and net purchases, indicating a prevalence of intangible, service-related activities. Simultaneously, these firms exhibit higher intangible and financial assets, as well as increased liquidity and profitability indexes.

Furthermore, Figure A1 shows the correlations between different types of degree centralities. It notably illustrates a strong correlation between per-worker out-degree and in-degree. This pattern indicates that firms facilitating significant worker outflows to various destinations also tend to receive inflows from numerous sources. I interpret these findings as a counterargument to the hypothesis that the per-worker out-degree measure merely reflects firms that workers are eager to leave due to poor conditions. The evidence of high inflow into these firms suggests otherwise, indicating their attractiveness to workers.

While my primary focus is on unweighted in- and out-degree centralities for their tractability and intuitive alignment with the model, I also incorporate other measures considering flow intensity in my supplementary results. Specifically, I evaluate two other degree centrality measures: weighted degree centrality and Opsahl. These additional measures are detailed in Appendix C. In Table A5, I provide different average degree centrality measures organized by industry. The average centrality ranking by industry remains largely stable across these measures. As expected, the most central firms are predominantly in the service sector (information and communications, financial services and insurance, accommodation and food services). Firms with the lowest centrality are typically found in heavy industries (mining and quarrying, water supply and waste management, transports) and education, where connectivity effects are arguably less crucial. This evidence is reported also by Figure **??**. Consistency is maintained when modifying the normalization criterion, transitioning from average to maximum employee count.

3.3 – Three empirical model's predictions

After outlining the key components used in this empirical study, I proceed to articulate three key predictions of the model to be tested against data.

Prediction 1. Workers should have shorter tenures in firms with higher connectivity.

This prediction directly stems from the core matching mechanism in my model. Workers who are matched with firms characterized by higher connectivity are more likely to receive external job offers, thereby increasing their chances of transitioning to a better match and reducing their tenure at the current firm.

Prediction 2. Workers should earn less when arriving at firms with higher connectivity. This negative relationship should be particularly evident at the beginning of a worker's career.

This is the implication of the interpretation of connectivity as a pure compensating differential which emerges from the wage equation (8). Workers starting at more connected firms are willing to accept lower current wages in exchange for enhanced future on-the-job search opportunities.

Prediction 3. Workers should earn more when leaving firms with higher connectivity.

This is interpreted as evidence of the faster climb of the job ladder guaranteed by firms with higher connectivity. Workers at these firms are more likely to receive offers from higher-paying, more productive companies, which would pay more. This should manifest in the data when comparing what happens to workers' salaries when they leave high-connectivity employers.

Figure 3 — Per-worker out-degree centrality and tenure

Notes. The figure illustrates the unconditional correlation between employee tenure and the per-worker out-degree at the firm level. Each point corresponds to the average within a specific ventile of the log per-worker out-degree. The red line represents the best fit from an OLS regression as detailed in Table A6. Source: Uniemens data, Istituto Nazionale della Previdenza Sociale (INPS).

In what follows, I will consider per-worker out-degree centrality as the main empirical proxy for a firm's connectivity.

3.4 – Firm-specific tenure and per-worker centrality

To validate Prediction 1 empirically, I run a regression that correlates firm-specific tenure at the end of a worker-firm match with the firm's per-worker out-degree centrality. Figure 3 shows the results using a log-log specification. It is obtained by grouping the data into 20 equal-sized bins based on log-centrality and calculating the average log-centrality and log-tenure for each bin. I then plot these averages against each other. The red line in the figure represents an OLS regression fit to these data points. I report the detailed regression results in Table A6.

The Prediction is confirmed, as the relationship is significantly negative, with a coefficient of -.12, meaning that a 10% increase in centrality is associated with a 1.2% decrease in firm-specific tenure.

Figure 4 — Per-worker out-degree centrality and starting earnings

Notes. The figure illustrates the unconditional correlation between employee salary in the first six months of the employment relationship and the per-worker out-degree at the firm level. Each point corresponds to the average within a specific ventile of the log per-worker out-degree. The red line represents the best fit from an OLS regression as detailed in Table A7. Source: Uniemens data, Istituto Nazionale della Previdenza Sociale (INPS).

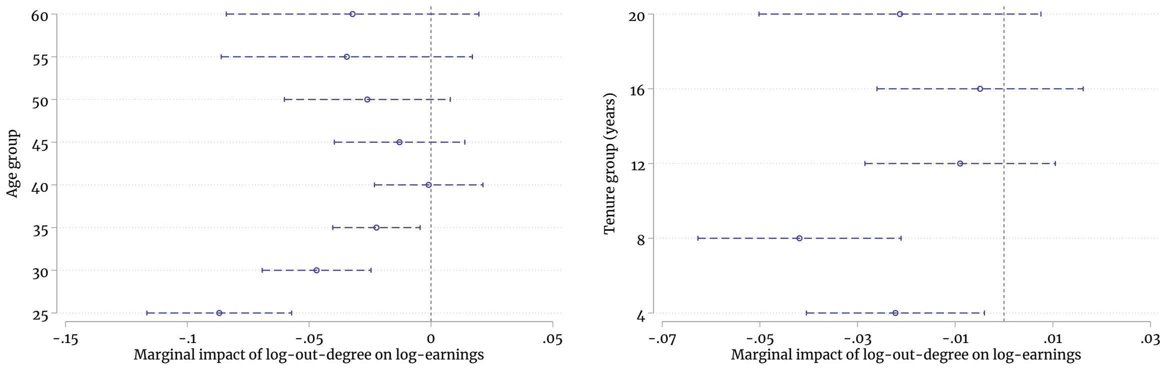
3.5 – Trading entry wage for connectivity

I now turn to document the presence of substantial heterogeneity in the firms' paying schemes at hiring, depending on their centrality and the age of the hired workers. This confirms the content of Prediction 2, and can be interpreted as evidence of the compensating differential mechanism described by my model.

Figure 4 replicates the same exercise of the previous Figure, this time considering as the dependent variable the log-earnings in the first 6 months of the match. Again, the relationship between the two is significantly negative. I estimate a coefficient of -.15, meaning that a 10% increase in the log of the per-worker out-degree centrality of the hiring firms is associated with a 1.5% decrease in hiring salary. Table A7 reports the detailed coefficients under different specifications as robustness exercises.

Still, since older workers (or workers with longer labor market experience) are more likely to be sorted into more-paying firms, this relationship is expected to be heterogeneous in age and tenure, by the same argument explaining the trade-off presented in Figure 2. To account

Figure 5 — Marginal relationship by age and tenure of per-worker out-degree centrality and starting earnings



(a) Age **(b) Labor market tenure**

Notes. The figure represents the coefficients $\beta_{2,j}$ from the specification detailed in (9). Specifically, the left panel (a) of the figure displays the coefficients for age groups, whereas the right panel (b) shows the coefficients for labor market tenure groups. Complete estimations can be found in Tables A8 and A9. The 95% confidence intervals are calculated by clustering at the firm-by-quarter level. Source: Uniemens data, Istituto Nazionale della Previdenza Sociale (INPS).

for this, I extend the model to include interaction terms between per-worker log out-degree centrality and eight age groups (20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59). In greater detail, I estimate

$$\log \text{Earnings}_{it} = \beta_1 \text{age}_{it} + \sum_j \beta_{2,j} \left(\text{age}^G_{it,j} \times \text{out-deg}_f \right) + \text{age}^G_j + \Phi_i + \tau_t + \varepsilon_{it} \tag{9}$$

where i and t index workers and quarters, respectively. Here, $\text{age}^G_{it,j}$ indicates a dummy variable that equals one if worker i is in age group j during quarter t; out-deg_t is the log per-worker out-degree of firm f where worker i is employed in t, age^G_j and τ_t represent age-group, and quarterly fixed effects, respectively, while Φ_i is a set of gender- and nationality-specific dummies. The term ε_{it} is the error component. I report the complete results of this augmented specification under different inclusions of fixed effects in Table A8. Figure 5 presents the coefficients $\beta_{2,j}$ from equation (9)—specifically the one reported in column (5) of Table A8. The right panel of the figure reports the same coefficients for a specification that considers labor market tenure groups (less than 4, 5-8, 9-12, 13-15, 16-20 years) rather than age ones. This alternative specification is reported in detail in Table A9. The data shows a strong negative correlation between centrality and initial earnings both for young and inexperienced workers, a correlation that diminishes as workers gain age and labor market experience. These findings align with the life cycle profile suggested by the job ladder mechanism in the model, where increased labor market tenure is associated with a higher likelihood of ascending the job ladder. Consequently, the compensating differential offered by firm connectivity is likely more significant for younger, inexperienced workers, given the greater room they face for sorting into better job positions.

3.6 – Is trading wage for connectivity worth it?

I now examine Prediction 3, which asks if sacrificing higher salaries for better job prospects—as discussed earlier—is truly beneficial.

A straightforward way to address this query is to compare the earnings of workers after they exit companies with high per-worker out-degree centrality against the earnings of those who leave more conventional companies. Doing so sheds light on the long-term advantages of accepting a lower starting salary at workplaces that provide increased job opportunities for their employees. A positive impact would clearly indicate that workers leaving these central firms ascend the job ladder more rapidly than workers leaving regular firms.

3.6.1 A data-driven procedure to identify high-connectivity firms

To delineate the relationship between future salaries and exiting a highly central firm, the sample of employers is divided into two distinct groups: high-connectivity and regular firms. This segregation is carried out using a k-means clustering algorithm applied to various degree centrality measures ascribed to firms.[10]

Specifically, I address the distance-minimization problem outlined as

$$\arg\min_{k_1,k_2} \sum_{i=1}^{K=2} \sum_{j \in k_i} \|\boldsymbol{C}(j) - \boldsymbol{\mu}_i\|^2 \tag{10}$$

Here, k_1 and k_2 represent the $K = 2$ clusters (highly central and regular firms), $\boldsymbol{C}(j)$ denotes the degree centrality vector for firm j, on which the algorithm clusters, and μ_i is the mean vector of centralities within cluster k_i. The distance between the centrality vector and the mean vector is calculated using the L-2 norm. The underlying rationale for this partitioning

10. The k-means algorithm is chosen for three primary reasons: a) It is an unsupervised learning algorithm, ensuring the procedure is entirely data-driven; b) Its simplicity and intuitive nature promote clear understanding; c) Its extensive application in social sciences (Steinley, 2006) and particularly in economics (for example, Bonhomme et al., 2019) make it a well-established choice.

process is to allocate each firm to a cluster in such a way that minimizes the within-cluster variance of the two centrality measures while maximizing the variance between clusters. Although k-means is an unsupervised algorithm, it presumes the number of partitions (in this case, $k = 2$) as a constant. Further reasoning supporting the choice for this two-group split, along with other insightful details on the process, can be found in Appendix D.

The primary clustering measures are the out-degree centrality alone and both in- and out-degree centrality. The preferred normalization is made by using the average firm size during the sample period. When clustering is based solely on out-degree centrality, the k-means algorithm divides the sample into two significantly different sizes: the high-connectivity firms account for 12.4% of the sample, while the remaining 87.6% fall under the regular category. Similar results are observed in the bi-dimensional clustering scenario, which also includes in-degree centrality, where the high-connectivity cluster comprises 15.2% of the employers in the sample.

3.6.2 How earnings change upon leaving a high-connectivity firm

I now turn to show that leaving a high-connectivity firm as the first job-to-job transition pays more than leaving a regular firm. To do so, I confront workers at a generic employment transition in their career in an "event study" setting in which treated units are employees leaving a high-connectivity firm, and controls are those leaving a regular firm, as previously defined. To do so, I rely on the Callaway and Sant'Anna (2021) estimator for cohort-specific relationships between residualized wages and the type of firm left. In particular, I compute the residuals $\hat{y}_{it} - y_{it}$ from the following model

$$\hat{y}_{it} = \alpha_i + \tau_t + \gamma_{s^O(i),t} + \eta_{s^D(i),t} + \psi_{p^O(i),t} + \lambda_{p^D(i),t} \tag{11}$$

where (α_i) represents individual fixed effects, τ_t are time (quarter) fixed effects, while $\gamma_{s^O(i),t}$ and $\psi_{p^O(i),t}$ account for specific time trends in the origin firm sector and province, respectively. Similarly, $\eta_{s^D(i),t}$ and $\lambda_{p^D(i),t}$ do the same for firms of destination. This specification allows me to control heterogeneous slopes in the firm- and province-specific time trends of both departure and arrival firms that might explain a relevant component of the variation in observed wages and earnings due to the job-to-job transition. The residuals are taken as differences between predicted values and either quarterly log-earnings of individual i at calendar time t

or their daily log-earnings.

I then employ the residuals in the estimation of the average treatment on workers leaving a high-connectivity firm, as per this specification

$$\text{ATT}(g, t; X) = \mathbb{E}\left[Y_{i,t} - Y_{i,g-1} | G_f = g\right] - \mathbb{E}\left[Y_{i,t} - Y_{i,g-1} | G_f \in \mathcal{G}\right] \tag{12}$$

where $\text{ATT}(g, t; X)$ is the average treatment effect at time t for the cohort of workers that moved out a high-connectivity firm in quarter g: for example, $\text{ATT}(2003q4, 2005q1)$ measures the impact of earnings in the first quarter of 2005 on the group of workers that have left a high-connectivity firm in the last quarter of 2003. $\mathcal{G}$ is the control group, defined as workers leaving regular firms within the same cohort.

To visualize the estimated impact on earnings in time deviation from the exit, it is a standard practice to aggregate the estimated treatments from (12) as weighted averages k periods away from the exit:

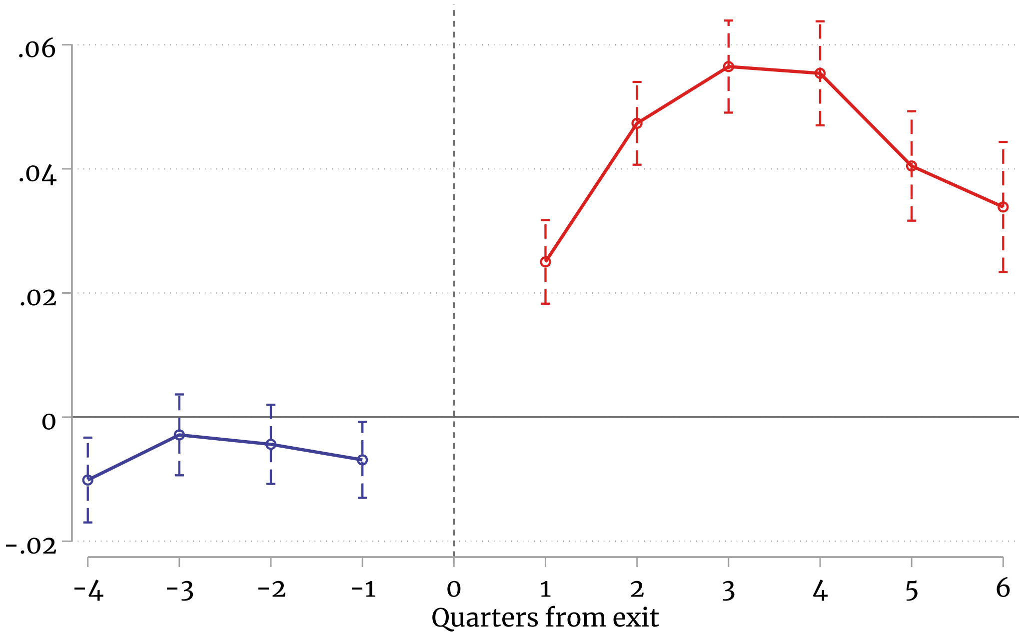
$$\text{ATT}_k = \sum_g w_g \text{ATT}(g, g + k) \tag{13}$$

where weights w weight cohorts for their relative frequencies in the treated population. Specifically, I consider the integers $k \in [-4, 6]$, thereby concentrating on a one to one-and-a-half-year window around the job movement.

Figure 6 shows the estimated *event study* coefficients computed in (13). Panel A shows the results for the specification using quarterly log earnings, while Panel B uses daily log earnings. Standard errors are clustered at the worker-by-quarter level. Overall, results suggest that leaving a high-connectivity firm pays off. Indeed, workers who move away from such employers earn, on average, around 5%-to-6% more than workers leaving regular firms after one year from the movement. Daily-earnings (Panel B) move similarly, showing a gap of up to 7% between the two groups of employees after slightly more than a year from the movement. To make sure that the estimated treatment does not stem from pre-existing differences between workers departing a "springboard" firm and those leaving a conventional firm, I run a Welch t-test on the average residualized earnings in the six months before the transition. The observed difference in levels between the two groups is 0.10%, with a corresponding p-value of 0.48, i.e., the disparity in residualized earnings between the groups is not statistically significant. Consequently, we can rule out significant pre-existing differences as a factor explaining the observed variation in earnings growth following the transition between the two groups.

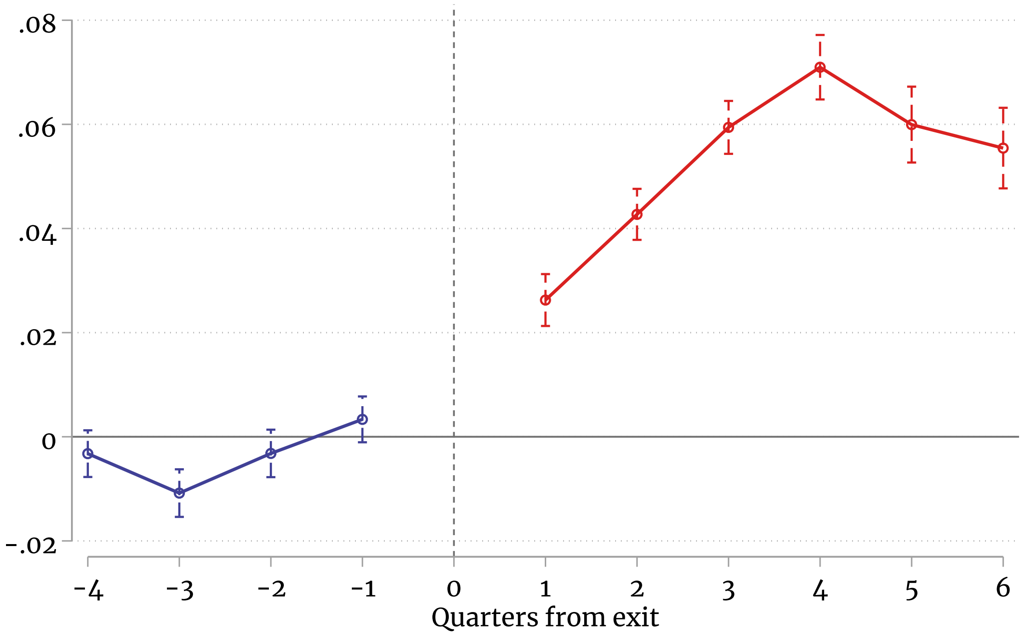
Figure 6 — Leaving a highly central firm vs. a regular one

(a) Quarterly earnings



Avg. relative difference in levels before leaving: 0.10%

(b) Daily earnings



Avg. relative difference in levels before leaving: -0.14%

Notes. These graphs show the relationship between leaving a high-centrality firm on a worker's quarterly log earnings (a) and log daily earnings (b). High-centrality firms are identified by solving the k-means problem (10) on per-worker out-degree centrality. Each point is the estimated ATT k quarters away from the first time a worker leaves a firm for an employer transition from (12). 95% confidence intervals are obtained by clustering at the individual-by-quarter level. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Of course, selection concerns make claiming causal identification in this specification difficult since it may be the case that workers with higher abilities systematically self-select into higher-connectivity firms. Still, results reported in Figures 4, 5 and 6 are notably coherent with the evidence of compensating differentials. They show that young workers accept lower wages when entering central firms in the job-to-job network for an investment of future value and that such investment is rebated when they leave those firms. The remainder of this paper is devoted to investigating further the economic mechanism behind these differences in wage dynamics.

3.7 – Human capital accumulation and signaling

A natural question that arises from the evidence of firms' heterogeneous contributions to job ladder climbing speed is whether this phenomenon is more closely related to human capital accumulation or to signaling effects. As in the seminal work by Spence (1973), which posits that workers may invest in education to signal their abilities to prospective employers, my conceptual framework is coherent with the possibility that some firms may be particularly effective in signaling the abilities of their workers. This signaling can contribute to an increase in the rate at which these workers receive external job offers. Concurrently, these same firms may also be proficient in facilitating a rapid increase in individual productivity among their employees. This, in turn, could provide a compelling reason for other firms to hire from them.

Traditionally, the question of how to account for wage differentials in terms of signaling versus human capital accumulation is tackled with education as the main signaling device (Weiss, 1995). However, in this work, I diverge from this classical perspective. Instead, I propose that past employment can also serve as a signaling mechanism, providing a different lens through which to assess a worker's quality.

In particular, I aim to decompose a past firm's effect on a worker's present wages among these two components. To achieve this, I start with a simple intuition: human capital accumulation is a function of the tenure a worker has had with a specific firm, whereas signaling is not necessarily tied to tenure. In terms of human capital accumulation, the longer an employee has worked for a particular firm, the more likely they are to have gained valuable skills, knowledge, or experience. This added human capital would subsequently be reflected in their higher wages (Gregory, 2023). On the other hand, the signaling effect is less dependent on firm-specific tenure. Once a worker has been employed by a firm, the signal of their ability and potential is sent to other employers, irrespective of how long the worker was actually employed there.

Building on this intuition, I leverage the tenure a worker has had at a previous firm as a means to evaluate its contributions in terms of either human capital accumulation or signaling effects. I do so by modifying the "dual wage ladder" AKM specification presented in Di Addario et al. (2023) to include an interaction term. This term captures tenure at the previous firm that is either strictly less than or greater than two years. By introducing this interaction term, the model can distinguish between the effects of short-term and long-term tenures at a past firm on a worker's current wages. For tenures shorter than two years, the signaling effect is likely to be more prominent. Conversely, human capital accumulation is expected to have a more significant impact on tenures longer than two years. In detail, I estimate the following extended AKM model:

$$\begin{aligned}\log(\text{earnings})_{it} = X'_{it} + \alpha_i + \overbrace{\psi_{j(i,t)}}^{\text{Destination's effect}} + \tau_t + \overbrace{\gamma_{h(i,t)} \times \mathbb{I}(\text{tenure}_{h(i,t)} < 2)}^{\text{Origin's signaling effect}} \\ + \underbrace{\gamma_{h(i,t)} \times \mathbb{I}(\text{tenure}_{h(i,t)} \geq 2)}_{\text{Origin's h.c. accumulation effect}} + \varepsilon_{it}\end{aligned} \tag{14}$$

Here, $\log(\text{earnings})_{it}$ represents the log earnings for worker i at time t, and X'_{it} is a vector of time-varying covariates, such as a second-degree polynomial for age.[11] The term α_i captures a worker-specific fixed effect, representing transferable abilities, while τ_t stands for a quarter-specific fixed effect. The term $\psi_{j(i,t)}$ represents the effect of the destination firm, and the terms $\gamma_{h(i,t)} \times \mathbb{I}(\text{tenure}_{h(i,t)} < 2)$ and $\gamma_{h(i,t)} \times \mathbb{I}(\text{tenure}_{h(i,t)} \geq 2)$ capture the signaling and human capital accumulation effects of the origin firm, respectively.

The model relies on the standard assumption of exogenous mobility, implying that the contribution of the past firm to current wages is not destination-specific, which is a requirement in line with my theoretical framework. In this setup, both $\psi_{j(i,t)}$ and $\gamma_{h(i,t)}$ are identified by instances where workers voluntarily change jobs.

Estimating the model (14) enables me to assign two separate fixed effects to each firm: one measuring their impact as an "origin" firm in terms of human capital accumulation, and the other in terms of signaling. Subsequently, I regress each of these fixed effects on the logarithm

11. I impose a linear restriction on age and time effects by normalizing the age profile at 40 years old. This circumvents the collinearity problems between date fixed effects and age controls highlighted, among others, in Card et al. (2013) and Card et al. (2018).

Figure 7 — Past firm's fixed effects, interacted by firm-specific tenure, against centrality

Notes. This figure illustrates the correlation between the interaction of estimated origin-firm fixed effects and firm-specific tenure, as defined by model 14, and per-worker out-degree centrality. An interaction term with firm-specific tenure of less than two years identifies the signaling effect of the originating firm on current earnings. On the other hand, interactions with longer tenures indicate human capital contributions. The figure includes lines that visually represent an OLS fit. Estimated coefficients corresponding to these relationships are detailed in Table A10. Each data point on the figure represents the average value within a specific bin of per-worker centrality. Source: Istituto Nazionale della Previdenza Sociale (INPS) and Cerved.

of the firm's average per-worker out-degree.

Figure 7 provides a graphical illustration of the results of this analysis, while the estimated coefficients are reported in Table A10. The fixed effects associated with short tenure at a worker's current firm exhibit a positive and statistically significant correlation with the firm's per-worker out-degree. This suggests that the greater the centrality of an origin firm, the greater its contribution to a worker's current wages via signaling effects. In contrast, the fixed effect capturing the firm's contribution to human capital accumulation shows no correlation with the firm's per-worker out-degree. This implies that the compensating differential offered by more centrally positioned firms is not attributable to higher rates of human capital accumulation at those firms.

4 — Estimation

After qualitatively analyzing the model and providing reduced-form evidence of its properties, I examine its quantitative impact on labor market dynamics. I estimate the model at a quarterly frequency on the same administrative dataset provided by INPS already detailed in Section 3.1. The estimation method used is *Indirect Inference*, an extension of the Simulated Method of Moments (McFadden, 1989). It employs auxiliary reduced-form specifications to refine the moments, aiming to minimize the discrepancy between data-derived and model-generated moments.[12] The details regarding the estimation procedure are presented and discussed in Appendix F.

Parametrization The model is fully parametrically estimated under some assumptions. First, as Jarosch (2023), I parametrize the marginal distributions governing firms' heterogeneity as betas: $\theta_p \sim \mathcal{B}(a_p, b_p)$ and $\theta_c \sim \mathcal{B}(a_r, b_r)$. I then allow their sampling to follow an empirical bivariate distribution governed by Frank's Copula $C(\varphi)$, where φ governs the covariance between the two dimensions of the firm in the job offer distribution. Moreover, I set the ability distribution as a standardized log-normal: $a \sim \log \chi(1, \sigma_a^2)$. For the numerical solution of the model, I approximate the employers' productivity and connectivity distributions on 25 points each, to obtain a grid of 625 distinct firm types. I similarly approximate workers' ability on 10 grid points. Therefore, numerically solving Equation (7), I build a multidimensional grid on which I will interpolate surplus value when simulating data. Moreover, the match's output is assumed additively separable: $f(a, \theta_p) = \kappa + a + \theta_p$ where κ is a location parameter common to all matches. I assume $\kappa = 1$. The model's period is quarterly, and workers participate in the labor market for 12 years (this matches the sample period I consider for the mobility network), implying 48 simulated periods. Finally, all along the estimation, I assume the model is in a steady state, i.e., workers' inflows and outflows across states are balanced.

4.1 – Identification

I proceed to discuss the identification of various model parameters that influence its outcomes. Although the parameters are estimated jointly, a heuristic discussion on how different sets of moments inform specific parameters can be valuable.

12. The foundational reference for indirect inference is Gourieroux et al. (1993). The method can be viewed as a generalized version of the Simulated Method of Moments (SMM).

Specifically, I address the issue of separately identifying the three components of wage dynamics: the worker-specific component, characterized by the distribution of ability $A(a)$, and the two firm-specific components, determined by the distributions $T(\theta_c)$ and $P(\theta_p)$ for connectivity and productivity, respectively. I also elaborate on how some features of the model can be linked to specific data metrics to differentiate between the two firm-specific components. Lastly, I introduce the moments that inform other standard parameters not directly related to the agents in the model. Table 1 displays, in the first two columns, the estimated parameters and brief descriptions, providing a quick overview of the parameter space.

4.1.1 Worker-specific determinants of wage heterogeneity

First, I want to separate the effects on wages of workers' heterogeneity from those of firms' heterogeneity to account for variation in workers' abilities. To do so, I adopt a two-way fixed effect specification (Abowd et al., 1999, AKM), running the following regression in the data

$$\log w_{it} = \alpha_i + \psi_{j(i,t)} + \gamma_t + X_{it}\beta + \varepsilon_{it} \tag{15}$$

Here, α_i, $\psi_{j(i,t)}$, and γ_t represent worker-, firm-, and time-fixed effects, respectively. The control variables X_{it} include second-degree polynomials for normalized age and qualification dummies.[13] I apply this regression specification to both the real and simulated data sets. For the simulated data, I use only time-varying controls related to experience, since other characteristics are not explicitly modeled.

It is well-recognized that the variances of fixed effect estimates can be upwardly biased due to limited worker mobility in both real and simulated data. This is because identification depends on workers changing jobs. Papers like Bonhomme et al. (2019) and Kline et al. (2020) offer correction methods, but these are computationally intensive when integrated into an SMM framework. Following the method by Gregory (2023), I mitigate this issue by reducing differences between real and simulated data, specifically by truncating workers' histories to reflect average job experience in the sample.

I run the regression in Equation (15) on real data for two consecutive sub-samples: 2006-2012 and 2012-2018. I perform a similar analysis on the simulated data, dividing it into two

13. Equation (15) is identified within connected components of the job-to-job network. As explained in section 3.2, my sample is already restricted to the largest connected component, covering 98.5% of employee transitions (Table A2). Thus, further data modifications for the connected-set requirement are unnecessary.

24-period groups to match the quarterly structure of the real data. I then target the firm-level variance of worker fixed effects for each sub-sample disregarding their correlation, as my model does not account for sorting across dimensions of heterogeneity.

4.1.2 Firm-specific determinants of wage heterogeneity

Connectivity. As discussed among the implications of the theoretical setting, the model predicts a job-to-job network where a firm's degree centrality maps into its connectivity. Therefore, I target different moments of the out-degree centrality to inform these parameters precisely. In particular, I match the variance and the interquartile range of both the per-worker out- and in-degree. This allows me to target the whole distribution of the two centralities. Moreover, the job-to-job (employer-to-employer, EE) rate is another crucial source of identifying the connectivity's distribution parameters since connectivity governs the meeting rate of firms—and thus, the likelihood of a movement to take place. Among the parameters informed by this set of moments, I also include the connectivity threshold I.

Productivity. I discipline the parameters governing the heterogeneity in the productivity of the firms targeting several wage moments, as in Bagger et al. (2014), Gregory (2023), and Jarosch (2023). In particular, I exploit wage changes between and within jobs: for the latter, I use the average wage change upon a job-to-job transition, while for the former, I use the average quarterly change in wages for stayers and the average wage change from the start to the end of a spell. Moreover, I target the interquartile range of the wage distribution. Clearly, these moments also convey information on the bargaining power parameter σ, which governs the magnitude of the wage responses both to employer changes and outside offers that lead to a renegotiation of the current compensation.

4.1.3 Other parameters

I exploit moments related to standard labor market flows to identify the job destruction rate. More specifically, since this parameter is exogenously set in the model, the unemployment-to-employment (EU) rate perfectly informs it. In particular, I calculate the period-specific rate and target its mean over the sample period. Since my dataset does not allow for observing unemployment-to-employment transitions directly—it is impossible to distinguish a worker in unemployment from one self-employed or working, for example, in the public sector—I

Table 1 — Estimated parameters and targeted moments

Parameter	Description	Estimate	Targeted moment(s)	Model	Data
		Panel A. Externally set or normalized			
β	Discount factor (quarterly)	97.5%	Herkenhoff et al. (2018)	-	-
λ	Job finding probability in unemployment	0.056	Gregory (2023)	-	-
κ	Production function location parameter	1.1	Jarosch (2023)	-	-
		Panel B. Internally estimated			
δ	Job destruction probability	0.7%	EU rate	0.007	0.007
σ	Workers' bargaining power	87.7%	Log wage IQR	0.48	0.6
			Log wage: P90-P50	0.59	0.66
a_p, b_p	Firm's productivity distribution	0.226, 0.280	Log wage: P50-P10	0.31	0.51
			Quarterly wage change	0.0001	0.0002
a_c, b_c	Firm's connectivity distribution	0.117, 2.335	Var. of per-worker out-degree	0.55	0.52
			IQR of per-worker out-degree	0.90	0.94
I	Connectivity threshold	0.607	Var. of per-worker in-degree	0.71	0.55
			IQR of per-worker in-degree	1.00	1.02
φ	Frank's copula parameter	4.56	EE rate	0.0094	0.01
			Btw-job wage change	0.09	0.09
σ_a^2	Worker's ability distribution	0.834	Workers FE var. (1st period)	0.094	0.097
			Workers FE var. (2nd period)	0.10	0.085

Notes. The table presents the outcomes of the model's structural estimation, alongside the targeted moments for calibration. Panel A lists the moments that are externally calibrated, while Panel B contains those that are internally estimated. All rates are reported on a quarterly basis. For a comprehensive explanation of these moments and how they inform the model's parameters, refer to Section 4.1. All parameters are estimated jointly. Further details on the estimation process can be found in Appendix F.

externally set the job-finding rate in unemployment. The workers' bargaining power, σ, is estimated together with the productivity distribution's parameters. Finally, as in Engbom (2020) and Gregory (2023), I externally set the discount factor, β, to a 3.75% quarter rate, following Herkenhoff et al. (2018).

4.2 – Estimates and model's fit

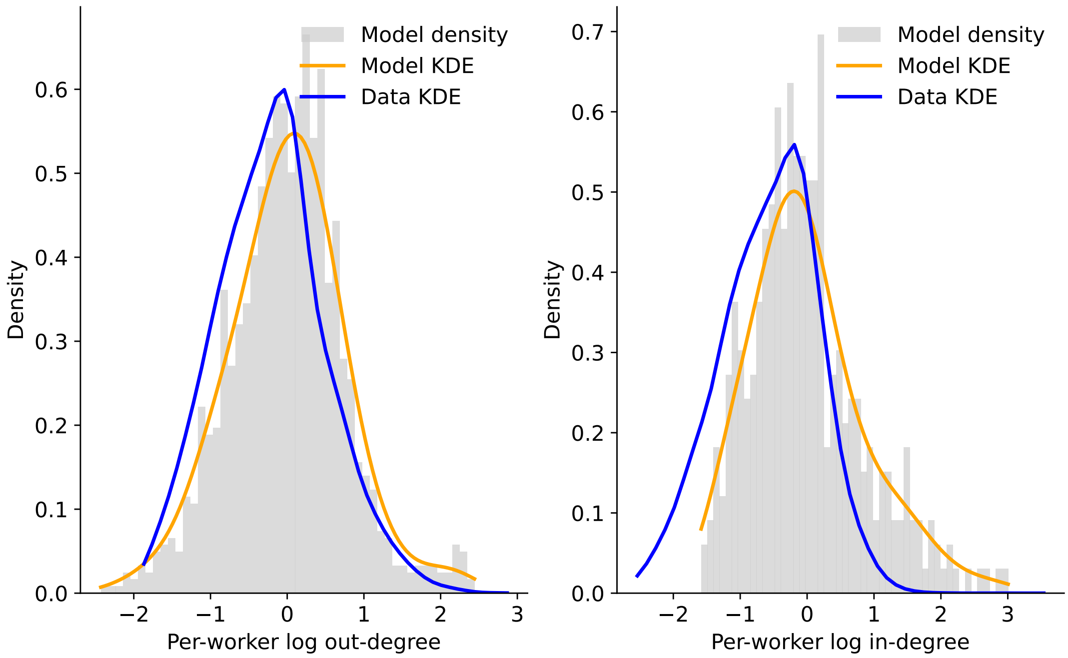
Table 1 offers a detailed summary of our estimation outcomes, listing both parameter estimates and their corresponding targeted moments from the model and actual data. Moments are grouped based on the identification criteria outlined in Subsection 4.1. However, all parameters are jointly estimated. Overall, the model exhibits a close fit to the data. The following paragraphs delve into the significance of these empirical moments and parameters in terms of model fit.

Flows. The targeted flows in the model, specifically EE and EU rates, align closely with the data. The EU rate directly informs the job destruction probability in the model, as job separations are exogenous. On the other hand, the EE rate is a more nuanced measure, reflecting the trade-offs workers make between productivity and connectivity in their job progression. It is noteworthy that the estimated connectivity threshold falls at the 99th percentile of the empirical connectivity distribution. To provide context, this threshold can be translated into an average probability of receiving a job offer—as it would be in a standard on-the-job random search model: $\lambda_E = \Pr\{\theta_c^1 + \theta_c^2 \geq I\}$ where θ_c^1 and θ_c^2 are random variables drawn from the estimated connectivity distribution. Monte Carlo simulations yield a quarterly probability of receiving a job offer of 0.024. This figure is comparatively lower than those in similar job-ladder models (Bagger et al., 2014; Krolikowski, 2017; Gregory, 2023; Jarosch, 2023, for estimates on US and Germany). While this could be indicative of the well-known less dynamic nature of the Italian labor market, it also highlights a key feature of my model: a limited number of firms facilitate a large volume of worker flows, thereby concealing significant heterogeneity.

Wages. I focus on two sets of wage-related moments for the estimation. The first set aims to capture wage dispersion through metrics such as the inter-quartile range of log wages and the differences between the 90th percentile and median, as well as between the median and the 10th percentile. The second set targets wage growth, both between jobs and over time. The model closely matches the upper end of the wage distribution but falls short of accurately capturing the lower end. This outcome suggests that my model's primary mechanism is more effective in describing top-wage earners than those at the bottom of the distribution. Additionally, the model performs well in representing between-job wage changes. However, it slightly underestimates constant quarterly wages, likely due to its omission of human capital accumulation dynamics.

Per-worker degrees. One of the key measures behind the core mechanism I investigate is the per-worker degree of a firm in the job-to-job network. Given Proposition 1, it is natural to target moments related to the dispersion of both per-worker out- and in-degree to inform connectivity. The model performs well in matching all the moments, falling slightly short on the variance of the in-degree, due to a small mismatch on the right tail. Yet, inter-quartile ranges are perfectly matched. Figure 8 shows the goodness of fit of the distribution of log

Figure 8 — Real data vs. model: distributions of log per-worker out- and in-degree

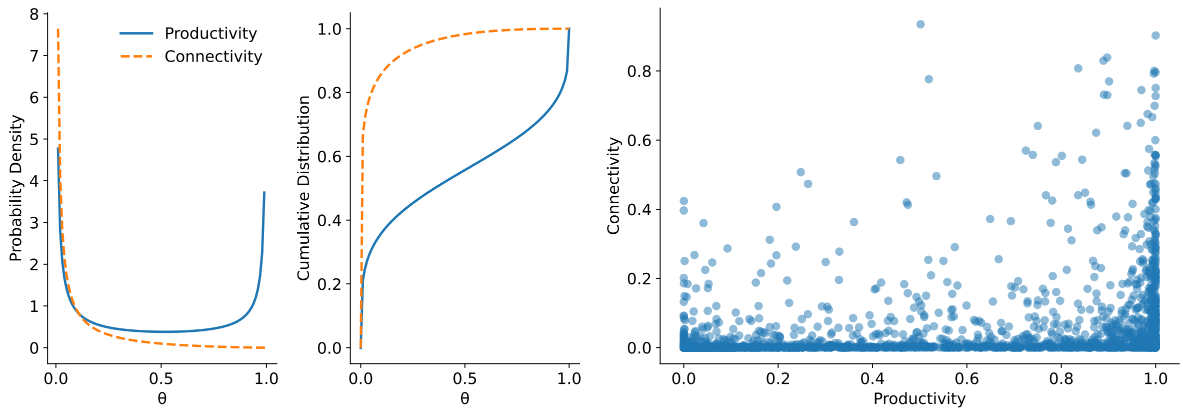


Notes. This plot displays the kernel density estimates for the mean-normalized log per-worker out-degree and in-degree, shown in the left and right panels, respectively.

per-worker out- and in-degree of the firms in the real and simulated job-to-job network.

Offer distribution. Figure 9 presents the estimated distributions of the two firm attributes: productivity and connectivity. The left panel displays both the probability and cumulative marginal densities, while the right panel shows a sample representing the empirical firm distribution, as governed by Frank's copula with two marginal Beta distributions. Firstly, the estimated value of φ is positive, indicating that more productive firms generally extend better outside job offers. This is in line with the empirical evidence concerning the correlation between per-worker out-degree and productivity, as shown in Figure A2. As for productivity, the pool from which workers sample has slightly more density at the lower end, which aligns closely with estimates from Jarosch (2023). In contrast, the distribution of connectivity is strongly right-skewed, a characteristic commonly observed in scale-free networks (Barabási and Albert, 1999). This further corroborates the model's connectivity parameter's alignment with the local structure of the mobility network.

Figure 9 — Estimated distributions of the firm productivity and connectivity



(a) Estimated distributions of firm attributes **(b) Estimated sample empirical distribution**

Notes. The figure presents the estimated attributes of the two firms. The left panel displays the estimated probability densities and cumulative densities for productivity (depicted in blue, continuous line) and connectivity (shown in orange, dashed line). The right panel provides an example of the actual distribution from which firms' attributes are sampled in the model. The correlation between productivity and connectivity is modeled using Frank's copula. The estimated parameters for the two beta distributions and Frank's copula are detailed in Table 1.

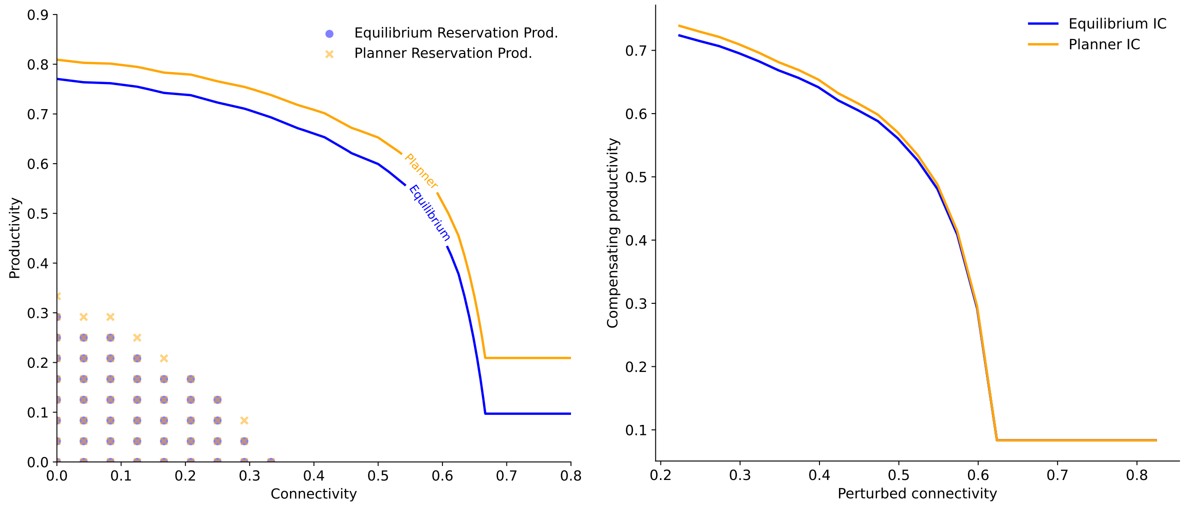
5 — Job ladder's efficiency

I now turn to the efficiency properties of job ladder climbing in the model. As pointed out by Jarosch (2023), in a standard partial equilibrium job ladder model with a single dimension of heterogeneity—productivity—the only decision margin prone to inefficiency is the reservation margin. In such a scenario, both a utilitarian planner maximizing welfare under search frictions and a worker ascending the job ladder concur that higher productivity jobs are superior. However, the current model introduces a trade-off for workers between connectivity and productivity, especially when a poaching firm doesn't dominate (or is dominated by) the current one in both dimensions. This prompts the question of whether the decentralized assignment of firms, along with these two attributes, to the job ladder rungs is efficient. In other words, in this context the planner has an added choice variable: the ability to rearrange the hierarchy of firms along the ladder based on both attributes.

The study of the utilitarian problem is particularly convenient in this case. First, due to linear preferences, maximizing welfare equates to maximizing the present discounted value of output. Second, the absence of congestion externalities and the exogeneity of job creation enable Pareto optimality when workers are granted full bargaining power.[14] This is formally

14. Notice that the steady state number of matches between workers and firms is $\lambda u + (1 -$

Figure 10 — Efficiency on the job ladder



(a) Surplus levels **(b) Indifference curves**

Notes. This figure illustrates efficiency in the job ladder. The left panel displays iso-quants of a fixed surplus value and the area of reservation jobs for a specific ability level. The right panel shows indifference curves (IC) between productivity and connectivity, indicating the level of productivity required to offset a change in connectivity. The planner's solution is determined by solving equation (7) with $\sigma = 1$. Reservation levels are pairs (θ_p, θ_c) such that $S(a, \theta_p, \theta_c) = 0$ in equation (7) for a given a. For additional details, refer to Section 5 and Appendix E.4.

shown in Appendix E.4. Intuitively, when the matching process doesn't involve any spillover costs or benefits to other parties in the market, workers with complete bargaining power fully internalize the value of their outside options, which would otherwise partly benefit firms.

Figure 10 presents the outcomes of the efficiency analysis. The left panel depicts iso-surplus curves for both the equilibrium and planner solutions. Both curves represent the same level of surplus generated by a match for a specific level of worker ability. Notably, the planner demands higher productivity levels across the board to achieve the same surplus. This is because, similarly to what happens in Jarosch (2023), in this setting a planner attributes a larger option value to search. Indeed, lower worker bargaining power reduces the scope for search—both during unemployment and on-the-job—since the room for improvement diminishes along with the share of surplus gained from a transition or renegotiation. In contrast, the planner fully internalizes the value of search and consequently places greater emphasis on a firm's connectivity over its productivity, as long as connectivity is a factor.

The right panel of Figure 10 clarifies this by illustrating the indifference curves that both workers and the planner face while keeping the surplus constant at the level shown in the left

$u) \int_{\underline{\theta}_c}^{\overline{\theta}_c} T(x) \left(\int_{I-x}^{\overline{\theta}_c} \mathrm{d}T(y) \right) \mathrm{d}x.$

panel. This curve is derived by solving for a specific surplus level and then calculating the productivity needed to offset a change in connectivity. Workers demand an inefficiently low level of productivity to make up for reduced connectivity, leading them to sort into firms that churn less than is socially optimal. This is further substantiated by the planner's adjusted reservation productivity (left panel), which extends the range that a job-seeking worker should consider acceptable when encountering a low-connectivity firm. In my framework, since search is more efficient during unemployment, the productivity required to offset the lost option value of searching in unemployment is higher in the social optimum than in equilibrium, making unemployed workers insufficiently selective regarding productivity.

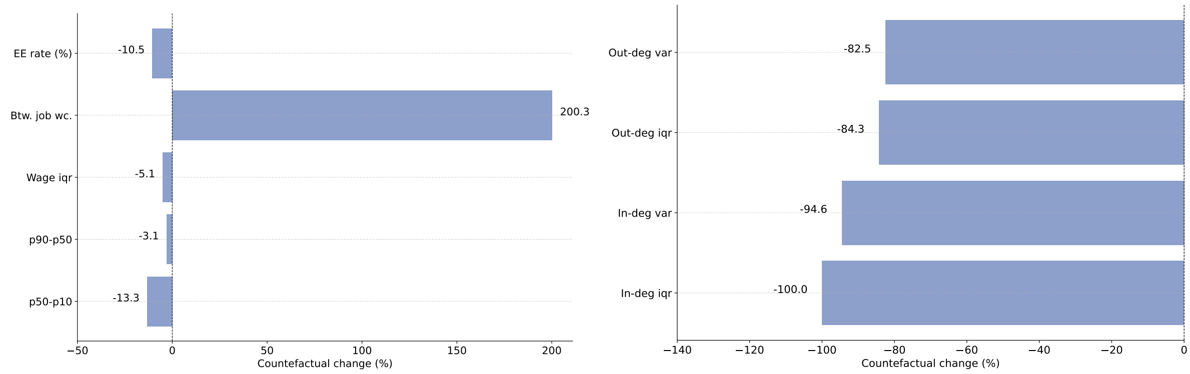
Although my framework omits key elements necessary for a comprehensive policy evaluation, the analysis remains instructive. It demonstrates the social importance of worker mobility and sheds light on the value lost due to inefficient firms with both low productivity and low connectivity, which trap workers in unproductive matches at the lower rungs of the job ladder.

6 — Counterfactuals

I now shift focus to assess the significance of the "connectivity channel" in explaining labor market dynamics. Specifically, I eliminate the primary source of heterogeneity in my model, which is the connectivity parameter. This simplification effectively reduces the model to a "standard" labor market framework where all employed workers have a uniform probability, denoted as λ_E, of receiving a job offer while engaged in on-the-job search. I set $\lambda_E \equiv \Pr\{\theta_c^1 + \theta_c^2 \geq I\}$, where θ_c^1 and θ_c^2 are random variables following the estimated distribution of connectivity. As previously detailed in Section 6, this probability is set at 0.024. Subsequently, I re-estimate the entire model, using the previously estimated parameters that govern the distribution of productivity and workers' ability, along with the estimated bargaining power.

The results of this exercise are summarized in Figure 11. The panels display the relative differences between key simulated moments in the homogeneous probability model and the full model. These differences capture the explanatory power of heterogeneous connectivity in my model for each moment considered.

Figure 11 — Counterfactuals: shutting down the main channel



(a) EE and wages **(b) Per-worker degrees**

Notes. This figure presents the outcomes of the counterfactual analysis discussed in Section 6. Each bar displays the relative difference between moments generated by the full model and those by a counterfactual model where employed workers have a uniform probability $\lambda_E = 0.024$ of receiving an external job offer. The left panel focuses on moments about worker transitions and wage distribution, whereas the right panel reports moments related to the per-worker degrees of firms in the job-to-job network.

Transitions and associated wage change. The left panel focuses on moments associated with transitions and wages. First, removing connectivity heterogeneity leads to a 10% decrease in simulated transitions, indicating that this specific type of firm heterogeneity accounts for one-tenth of observed transitions. Additionally, the simulated average wage for a transition in the counterfactual scenario is almost threefold compared to the full model. This occurs because the counterfactual overlooks the crucial role of connectivity in shaping the compensating differentials faced by workers. Figure A3 illustrates the impact on wage change distributions in both models. Two key observations can be made. First, in the counterfactual, transitions never occur with a wage reduction. This is because workers' estimated bargaining power is large enough to allow productivity's present-value contribution to overshadow that of compensating differentials in the wage equation (8). Second, the distribution of wage changes is more uniform in the counterfactual, increasing the prevalence of transitions associated with larger wage hikes compared to the full model.

Wage distribution. The counterfactual model is less accurate in predicting the interquartile range of the overall wage distribution and the inequality at its upper end. Although the discrepancy is small, it is consistently negative across all moments. This aligns with the simple intuition that equalizing job offer probabilities reduces overall wage inequality. Yet, this effect is particularly pronounced at the lower end of the wage distribution. Shutting down the heterogeneity in connectivity results in a 13% underestimation of the gap between the median wage and its tenth percentile. This suggests that the lower tail of the wage distribution is more sensitive to my mechanism. This can be attributed to ”inequality pockets” that my model generates, where workers remain stuck in low-productivity, low-connectivity jobs for extended periods. In the counterfactual setting, on the contrary, workers at the lower wage levels have equal chances of moving to higher-paying jobs as those at or above the median, thus reducing inequality at the lower end.

Per-worker degrees. The left panel of Figure 11 illustrates changes in the moments that describe the distribution of per-worker in- and out-degree. The large decrease in both the variance and interquartile range of these measures in the counterfactual scenario points to a more uniform labor market. In this simplified setting, firms exhibit the same levels of connectivity, thus reducing the extremes of highly connected and highly isolated firms. This structural shift has repercussions for worker mobility, consistent with the earlier observed narrowing of the lower end of the wage distribution. Absent significant ”springboards” to pull in or push out a large number of workers, job transition opportunities become more evenly spread across the labor market. This change may particularly benefit workers in the lower half of the wage distribution by increasing their likelihood of finding better jobs. Moreover, the counterfactual model suggests alterations in job-matching dynamics that align with the observed reduction in simulated transitions. This homogeneity could enhance search efficiency, potentially reducing the number of job switches that would otherwise occur in a heterogeneous setting due to broader future opportunities.

7 — Conclusions

This paper integrates the concept of a firm’s ”springboard” potential into a labor market search model, examining its impact on individual career progression and wage inequality. Firms vary in their ability to generate external job offers for their employees, creating a trade-off for workers between productivity and connectivity, the latter being the firm-specific factor influencing the rate of external job offers. Per-worker out-degree centrality in the job-to-job network acts as a good empirical measure of this characteristic. It positively correlates with key financial metrics, indicating that firms providing better opportunities are often profitable and productive. It also aligns with the model’s core predictions, revealing both a premium and a compensating differential that workers experience due to the "springboard" mechanism. Quantitatively, the model effectively captures targeted aspects of wage distribution, job-to-job network, and transitions. The firm-induced variation I introduce accounts for 10% of observed job changes and is a significant driver of inequality at the bottom of the wage distribution, as it contributes to trapping workers in low-productivity firms.

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A — Additional figures and tables

Table A1 — Descriptive statistics of the transitions in the panel

	2008-2011	2011-2014	2014-2017	2017-2020	2006-2020 (whole period)
Demographics					
Sh. of movements made by females	0.33	0.32	0.32	0.32	0.32
	(0.471)	(0.465)	(0.467)	(0.467)	(0.468)
Sh. of movements made by Italians	0.94	0.94	0.95	0.95	0.95
	(0.236)	(0.228)	(0.220)	(0.224)	(0.227)
Tenure when leaving	1.47	2.58	3.23	3.59	2.66
	(1.022)	(1.918)	(2.891)	(3.492)	(2.671)
Avg. age at					
generic movement	35.0	35.9	36.4	36.6	35.9
	(8.904)	(9.370)	(9.756)	(10.03)	(9.530)
first movement	34.7	35.0	34.6	34.3	34.6
	(8.933)	(9.411)	(9.718)	(9.860)	(9.458)
Share of movements within the same					
2-digits industry	0.34	0.32	0.30	0.29	0.31
	(0.473)	(0.467)	(0.459)	(0.455)	(0.464)
province	0.44	0.41	0.40	0.38	0.41
	(0.496)	(0.492)	(0.489)	(0.486)	(0.492)
Earnings					
when leaving	8432.9	8328.9	8432.8	8571.5	8480.2
	(6367.4)	(6551.9)	(6470.4)	(6171.3)	(6394.4)
when arriving	8148.7	7902.0	8207.3	8484.3	8228.8
	(6303.1)	(6376.8)	(6354.3)	(6133.5)	(6312.6)
Pct. difference	9.26	8.47	11.2	10.4	9.69
	(80.01)	(84.77)	(82.52)	(73.59)	(79.53)
Wages					
when leaving	8340.3	7008.1	7060.0	7102.7	7505.5
	(269624.6)	(14074.6)	(9318.0)	(6802.3)	(156659.3)
when arriving	7428.7	6934.8	7214.0	7461.1	7322.8
	(106060.1)	(11812.8)	(8486.1)	(7144.9)	(61894.6)
Pct. difference	6.97	4.35	8.20	9.74	7.50
	(59.98)	(57.45)	(56.98)	(51.68)	(56.30)
Movers	425948	358650	376454	345192	1264302

Notes. The table reports selected descriptive statistics regarding the job-to-job transitions in the panel. Each column reports summaries for a four-year subperiod, while the last one considers the entire time sample. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A2 — Nodes and links in the job-to-job network

	2008-2011	2011-2014	2014-2017	2017-2020	2008-2020 (complete)
Number of nodes	120,121	111,555	105,651	90,241	198,036
Number of links	965,195	886,608	870,499	676,010	1,936,383
Number of connected components	8,210	8,732	7,658	6,390	7,709
In the largest connected component					
% of nodes	84.6	82.1	83.5	84.1	91.1
% of links	98.3	97.8	98.0	98.0	99.2

Notes. Number of nodes and links in the job-to-job network, between 2006 and 2020, with four sub-periods breakdown. Each node is a firm that faced at least one employment transition in the sample period and each link is a transition. The largest connected component is the maximal set of nodes such that each pair of them is connected by a path, which accounts for nearly all the observed movements between firms. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A3 — Descriptive statistics of the panel

	Whole panel Q2 2010		Largest CC Q2 2010		Whole panel Q2 2014		Largest CC Q2 2014		Whole panel Q2 2018		Largest CC Q2 2018	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Size	14.22	179.37	21.39	227.10	15.61	325.92	24.21	420.24	16.37	320.97	28.67	446.66
Firm age	15.28	12.52	14.60	12.56	17.02	13.43	16.52	13.40	18.71	14.36	20.46	13.96
Sh. in manufacturing	0.67	0.47	0.64	0.48	0.66	0.47	0.63	0.48	0.65	0.48	0.63	0.48
Sh. in services	0.28	0.45	0.32	0.47	0.30	0.46	0.33	0.47	0.31	0.46	0.33	0.47
Sh. of female workers	0.35	0.48	0.34	0.47	0.36	0.48	0.36	0.48	0.37	0.48	0.36	0.48
Sh. of Italian workers	0.97	0.18	0.97	0.18	0.97	0.18	0.97	0.18	0.96	0.19	0.96	0.19
Sh. of under 35	0.35	0.48	0.34	0.48	0.25	0.43	0.25	0.43	0.22	0.42	0.22	0.41
Sh. of 36-55	0.59	0.49	0.60	0.49	0.65	0.48	0.65	0.48	0.62	0.48	0.63	0.48
Sh. of over 55	0.06	0.24	0.06	0.24	0.10	0.30	0.10	0.30	0.15	0.36	0.16	0.36
Avg. firm-specific tenure	2.17	0.65	2.17	0.64	4.77	2.16	4.77	2.13	6.49	3.90	6.65	3.84
Avg. labor market experience	16.61	9.96	16.62	9.95	19.02	10.33	19.06	10.32	20.33	11.21	20.46	11.17
Quarterly earnings	3,657.38	2,280.66	3,747.37	2,302.89	3,511.64	2,168.98	3,590.41	2,186.91	3,647.84	2,267.67	3,742.04	2,289.72
Workers	2011742		1861221		2090010		1937831		2201817		1978551	
Firms	141487		87007		133905		80057		134491		69021	

Notes. The table reports selected descriptive statistics for the whole panel (first column) and the largest connected component in the job-to-job network (second column). Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A4 — Descriptive statistics of the worker-firm panel, by quartiles of normalized centrality

Normalized centrality quartile	Age	Age at hiring	Quarterly wage	Firm-specific tenure (quarters)	Females	Italians	Num. of workers	Num. of firms
1	45.41 (9.49)	42.15 (9.48)	10,243 (5,976)	35.50 (15.73)	35.4%	98.0%	933,710	44,897
2	42.04 (9.52)	39.64 (9.68)	10,292 (6,475)	36.11 (15.48)	34.1%	96.3%	1,019,827	44,896
3	40.80 (9.45)	38.43 (9.20)	10,608 (7,036)	35.05 (16.25)	34.7%	96.3%	799,927	44,896
4	39.86 (9.15)	38.04 (8.88)	11,443 (7,884)	33.58 (16.09)	32.1%	96.7%	979,254	44,896
Entire Panel	42.08 (9.64)	39.62 (9.46)	10,654 (6,896)	35.06 (15.89)	33.9%	96.8%	2,742,853	179,585

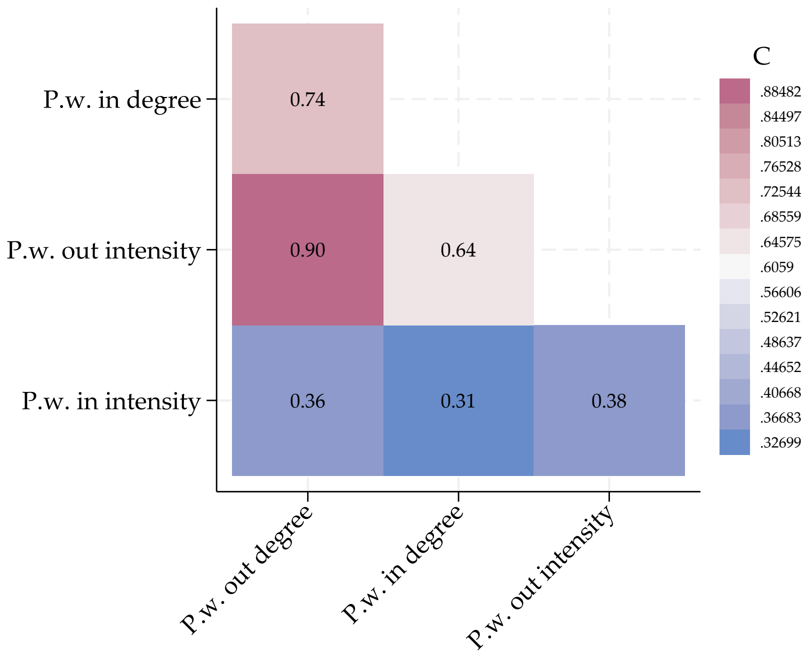
Notes. The table reports firm-level means and standard deviation (in parenthesis) of selected measures by quartiles of out-degree centrality, normalized by the average firm's size. The panel comprises full-time, white-collar workers in Italian private large firms with at least one job-to-job transition. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A5 — Degree centrality by industry

	Mining & quarrying	Manufacturing	Electricity/ gas/ steam/AC	Water supply/ sewerage/waste	Constructions	Wholesale/ retail trade	Transporting & storage	Accomodation/ food service	Information & communication	Finance/ insurance	Real estate	Professional/ technical activities	Administration/ support service	PA/ defence	Education	Health and Social work	Arts/ entertainment	Other services	Households	Extraterritorial organizations	Total
Normalization by avg. size																					
Out unw.	0.045	0.048	0.065	0.035	0.051	0.059	0.044	0.052	0.11	0.075	0.092	0.090	0.055	0.029	0.041	0.036	0.055	0.046	0.064	0.032	0.055
Out weighted	0.061	0.056	0.088	0.053	0.060	0.071	0.059	0.056	0.15	0.11	0.12	0.12	0.073	0.067	0.057	0.049	0.065	0.062	0.087	0.039	0.068
Out Opsahl	0.051	0.051	0.074	0.041	0.055	0.063	0.050	0.054	0.13	0.088	0.10	0.10	0.062	0.041	0.046	0.040	0.058	0.051	0.072	0.035	0.060
In unw.	0.070	0.081	0.13	0.072	0.086	0.11	0.080	0.063	0.17	0.13	0.16	0.15	0.095	0.049	0.077	0.064	0.088	0.077	0.097	0.088	0.092
In weighted	0.22	0.21	0.45	0.21	0.13	0.28	0.19	0.077	0.68	0.62	0.46	0.53	0.26	0.34	0.35	0.28	0.42	0.26	0.14	0.66	0.26
In Opsahl	0.10	0.11	0.22	0.11	0.084	0.15	0.10	0.053	0.32	0.26	0.24	0.26	0.14	0.12	0.15	0.12	0.16	0.12	0.093	0.22	0.13
Normalization by max size																					
Out unw.	0.029	0.033	0.042	0.022	0.030	0.039	0.028	0.032	0.070	0.050	0.050	0.057	0.034	0.019	0.026	0.025	0.032	0.030	0.040	0.024	0.036
Out weighted	0.038	0.038	0.057	0.031	0.035	0.046	0.037	0.034	0.096	0.074	0.064	0.074	0.044	0.043	0.036	0.033	0.038	0.040	0.050	0.029	0.044
Out Opsahl	0.032	0.035	0.048	0.025	0.032	0.042	0.031	0.033	0.080	0.058	0.055	0.063	0.038	0.026	0.029	0.028	0.034	0.034	0.044	0.026	0.039
In unw.	0.048	0.056	0.087	0.047	0.050	0.071	0.051	0.039	0.10	0.090	0.090	0.093	0.058	0.032	0.051	0.044	0.051	0.051	0.065	0.069	0.060
In weighted	0.15	0.15	0.32	0.14	0.081	0.19	0.13	0.050	0.42	0.45	0.27	0.34	0.17	0.22	0.23	0.21	0.23	0.18	0.10	0.51	0.18
In Opsahl	0.073	0.081	0.15	0.070	0.051	0.10	0.068	0.034	0.19	0.18	0.14	0.16	0.086	0.075	0.098	0.082	0.090	0.084	0.066	0.18	0.089

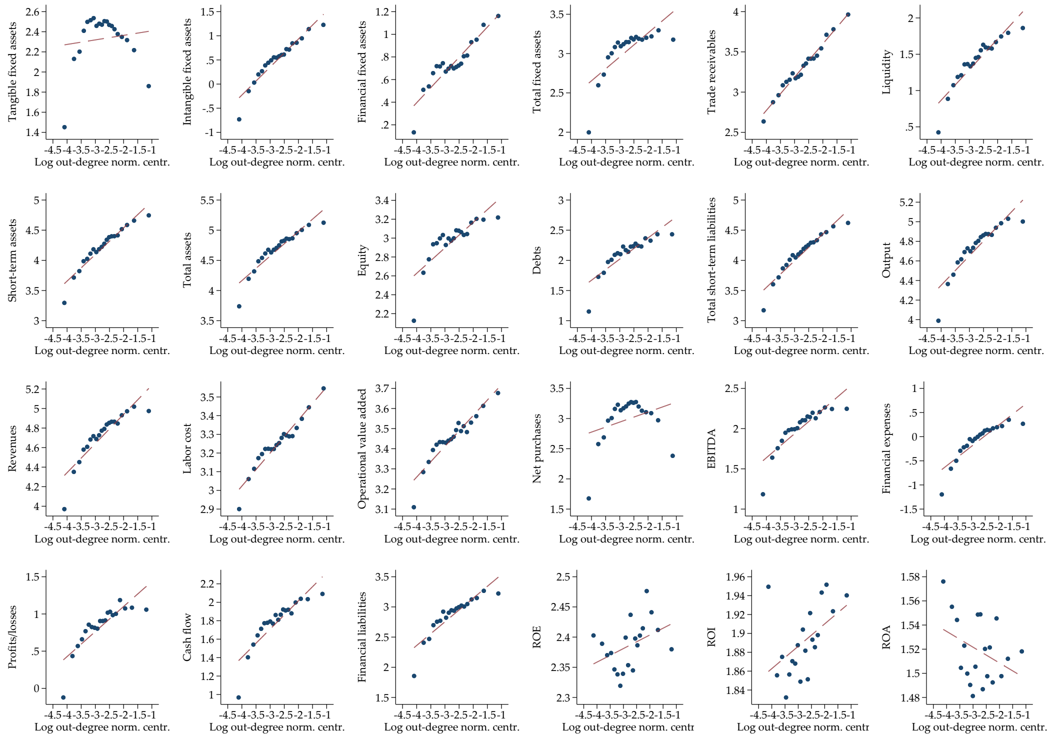
Notes. The table reports average centrality measures by 2-digit industries. The top panel normalizes centrality by the firm's average number of employees over the period; the bottom panel does so by its maximum. Each panel reports the following measures, in order: out-degree unweighted centrality, out-degree weighted centrality, out-degree Opsahl centrality, in-degree unweighted centrality, in-degree weighted centrality, in-degree Opsahl centrality. Opsahl centrality assumes $\alpha = 0.5$. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Figure A1 — Correlation matrix of per-worker centralities

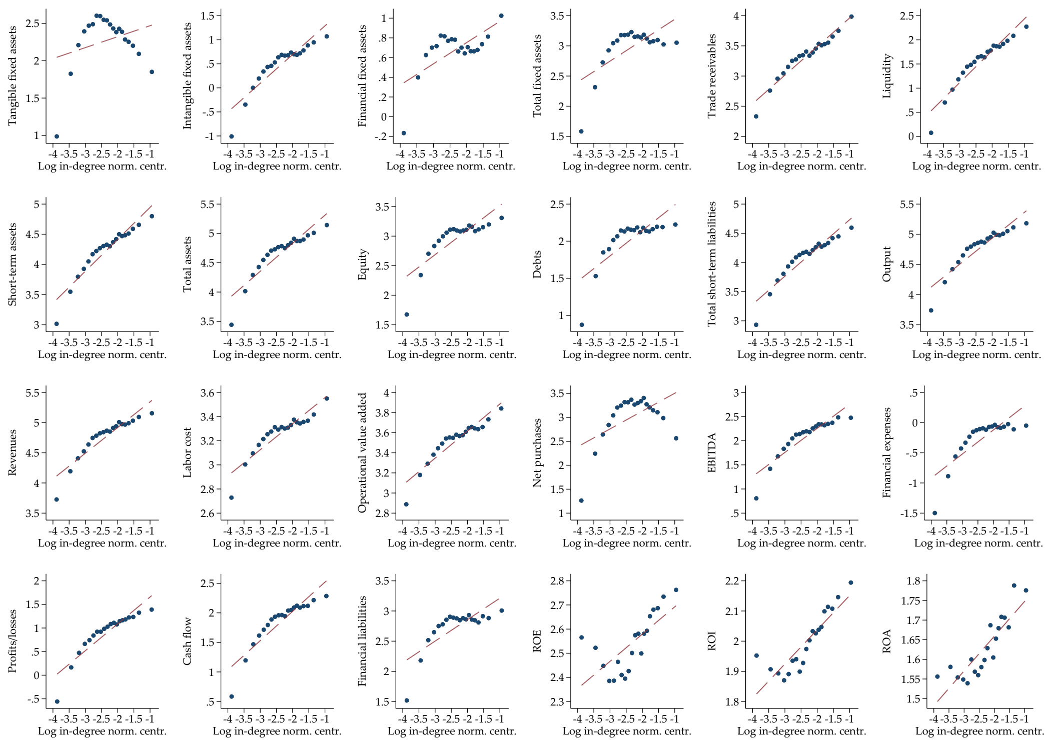


Notes. The entries in the diagonal matrix represent the correlations between centralities in the firm sample. Per-worker degree is calculated as the ratio of the number of neighbors in the directed network to the average firm size (number of workers) over the 2008-2018 period. Per-worker intensity is defined as the ratio of the number of directed flows to the average firm size. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Figure A2 — Firm-level financial measures by degree centrality



(a) By per-worker out-degree



(b) By per-worker in-degree

Notes. This figure plots the means of relevant financial variables at the firm level over 2008-2018, by ventiles of per worker out-degree (Panel A) and in-degree (Panel B). Source: Istituto Nazionale della Previdenza Sociale (INPS) and Cerved.

Table A6 — Relationship between firm-specific tenure and per-worker out-degree centrality

	(1) log tenure	(2) log tenure	(3) log tenure	(4) log tenure	(5) log tenure
Log per-worker out-degree	-0.132***	-0.132***	-0.129***	-0.104***	-0.0886***
	(-74.52)	(-74.57)	(-73.30)	(-61.32)	(-48.62)
Female fixed effects		✓	✓	✓	✓
Italian fixed effects			✓	✓	✓
Year fixed effects				✓	✓
Province fixed effects					✓
Observations	1374188	1374188	1374188	1374188	1374188

Notes. This table presents the estimated coefficients from an OLS analysis, where log firm-specific tenure is regressed on log per-worker out-degree, under different fixed effects inclusions. The 95% standard errors are clustered at the firm level. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A7 — Relationship between earnings at entry and per-worker out-degree centrality

	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	8.208***	4.423***										
	(0.0421)	(0.0272)										
Log out-degree	-0.1499***	-0.0633***	-0.1500***	-0.0635***	-0.1473***	-0.0630***	-0.1492***	-0.0630***	-0.0618***	-0.0405***	-0.0622***	-0.0464***
	(0.0164)	(0.0106)	(0.0167)	(0.0103)	(0.0162)	(0.0102)	(0.0168)	(0.0102)	(0.0125)	(0.0096)	(0.0107)	(0.0083)
Observations	3,280,544	3,212,458	3,280,544	3,212,458	3,280,544	3,212,458	3,280,544	3,212,458	3,280,544	3,212,458	3,280,544	3,212,458
R^2	0.01474	0.00900	0.04016	0.05005	0.04299	0.05038	0.04516	0.05134	0.30678	0.13131	0.32150	0.15505
Within R^2			0.01515	0.00944	0.01463	0.00928	0.01502	0.00928	0.00343	0.00401	0.00329	0.00500
Female fixed effects			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Italian fixed effects					✓	✓	✓	✓	✓	✓	✓	✓
Past spells fixed effects							✓	✓	✓	✓	✓	✓
Starting date fixed effects									✓	✓	✓	✓
Province fixed effects											✓	✓

Notes. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A8 — The relationship between initial earnings and per-worker out-degree centrality, by age

	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings	Log earnings	Log daily earnings
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
age	0.0536***	0.0266***	0.0529***	0.0262***	0.0528***	0.0262***	0.0529***	0.0262***	0.0357***	0.0238***	0.0362***	0.0244***
	(0.0020)	(0.0012)	(0.0019)	(0.0011)	(0.0019)	(0.0011)	(0.0019)	(0.0011)	(0.0011)	(0.0007)	(0.0011)	(0.0007)
Log out-degree × 20-24 y.o.	-0.0866***	-0.0295***	-0.0872***	-0.0301***	-0.0869***	-0.0300***	-0.0817***	-0.0301***	-0.0390***	-0.0229***	-0.0502***	-0.0365***
	(0.0158)	(0.0066)	(0.0153)	(0.0064)	(0.0152)	(0.0064)	(0.0156)	(0.0065)	(0.0103)	(0.0057)	(0.0099)	(0.0055)
Log out-degree × 25-29 y.o.	-0.0457***	-0.0176***	-0.0476***	-0.0189***	-0.0469***	-0.0188***	-0.0445***	-0.0191***	0.0045	-0.0108**	-0.0102*	-0.0258***
	(0.0119)	(0.0054)	(0.0115)	(0.0052)	(0.0114)	(0.0052)	(0.0118)	(0.0052)	(0.0067)	(0.0046)	(0.0062)	(0.0044)
Log out-degree × 30-34 y.o.	-0.0208**	0.0009	-0.0236**	-0.0010	-0.0224**	-0.0010	-0.0226**	-0.0011	-0.0009	-3.39×10^{-5}	-0.0119	-0.0114*
	(0.0095)	(0.0063)	(0.0094)	(0.0060)	(0.0092)	(0.0060)	(0.0095)	(0.0060)	(0.0081)	(0.0067)	(0.0073)	(0.0062)
Log out-degree × 35-39 y.o.	0.0014	0.0228**	-0.0028	0.0197*	-0.0009	0.0198*	-0.0023	0.0199**	0.0147	0.0190*	0.0048	0.0076
	(0.0119)	(0.0105)	(0.0115)	(0.0101)	(0.0114)	(0.0101)	(0.0114)	(0.0100)	(0.0104)	(0.0098)	(0.0091)	(0.0080)
Log out-degree × 40-44 y.o.	-0.0126	0.0215	-0.0156	0.0193	-0.0129	0.0195	-0.0155	0.0202	0.0092	0.0200	0.0019	0.0091
	(0.0143)	(0.0140)	(0.0138)	(0.0137)	(0.0137)	(0.0137)	(0.0135)	(0.0134)	(0.0135)	(0.0129)	(0.0119)	(0.0103)
Log out-degree × 45-49 y.o.	-0.0241	0.0220	-0.0294*	0.0181	-0.0261	0.0183	-0.0309*	0.0194	-0.0067	0.0153	-0.0116	0.0053
	(0.0192)	(0.0194)	(0.0174)	(0.0178)	(0.0174)	(0.0178)	(0.0170)	(0.0173)	(0.0154)	(0.0138)	(0.0131)	(0.0108)
Log out-degree × 50-54 y.o.	-0.0295	0.0183	-0.0374	0.0124	-0.0345	0.0126	-0.0401	0.0138	-0.0386*	-0.0014	-0.0383**	-0.0076
	(0.0295)	(0.0319)	(0.0261)	(0.0291)	(0.0263)	(0.0291)	(0.0256)	(0.0286)	(0.0221)	(0.0193)	(0.0193)	(0.0153)
Log out-degree × 55-59 y.o.	-0.0242	0.0249	-0.0348	0.0171	-0.0322	0.0173	-0.0366	0.0185	-0.0509*	0.0019	-0.0465*	-0.0025
	(0.0290)	(0.0291)	(0.0263)	(0.0270)	(0.0264)	(0.0269)	(0.0258)	(0.0265)	(0.0281)	(0.0169)	(0.0260)	(0.0141)
Observations	3,185,014	3,125,541	3,185,014	3,125,541	3,185,014	3,125,541	3,185,014	3,125,541	3,185,014	3,125,541	3,185,014	3,125,541
R^2	0.23041	0.21584	0.24553	0.24271	0.24750	0.24274	0.24998	0.24336	0.36997	0.25996	0.38417	0.28630
Within R^2	0.00912	0.00720	0.00929	0.00709	0.00920	0.00709	0.00923	0.00717	0.00466	0.00577	0.00498	0.00658
Age group fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Female fixed effects			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Italian fixed effects					✓	✓	✓	✓	✓	✓	✓	✓
Past spells fixed effects							✓	✓	✓	✓	✓	✓
Starting date fixed effects									✓	✓	✓	✓
Province fixed effects											✓	✓

Notes. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A9 — The relationship between initial earnings and per-worker out-degree centrality, by tenure

	Log earnings (1)	Log daily earnings (2)	Log earnings (3)	Log daily earnings (4)	Log earnings (5)	Log daily earnings (6)	Log earnings (7)	Log daily earnings (8)	Log earnings (9)	Log daily earnings (10)	Log earnings (11)	Log daily earnings (12)
tenure	0.1308***	0.0353***	0.1292***	0.0341***	0.1295***	0.0348***	0.1222***	0.0336***	0.0374***	0.0201***	0.0368***	0.0201***
	(0.0025)	(0.0010)	(0.0024)	(0.0010)	(0.0024)	(0.0010)	(0.0023)	(0.0011)	(0.0016)	(0.0008)	(0.0015)	(0.0008)
Log out-degree × 0-3 years	-0.0180*	-0.0105***	-0.0223**	-0.0137***	-0.0222**	-0.0136***	-0.0156	-0.0126***	0.0018	-0.0094**	-0.0107*	-0.0220***
	(0.0092)	(0.0038)	(0.0093)	(0.0039)	(0.0093)	(0.0039)	(0.0097)	(0.0040)	(0.0071)	(0.0037)	(0.0064)	(0.0034)
Log out-degree × 4-7 years	-0.0401***	-0.0060	-0.0418***	-0.0071	-0.0419***	-0.0073	-0.0400***	-0.0071	-0.0165*	-0.0037	-0.0251***	-0.0132**
	(0.0108)	(0.0072)	(0.0106)	(0.0070)	(0.0106)	(0.0071)	(0.0113)	(0.0073)	(0.0091)	(0.0070)	(0.0072)	(0.0058)
Log out-degree × 8-11 years	-0.0056	0.0150*	-0.0088	0.0129*	-0.0090	0.0126*	-0.0096	0.0125*	0.0059	0.0139*	-0.0020	0.0046
	(0.0102)	(0.0078)	(0.0099)	(0.0074)	(0.0100)	(0.0075)	(0.0103)	(0.0075)	(0.0091)	(0.0073)	(0.0078)	(0.0063)
Log out-degree × 12-15 years	-0.0013	0.0158	-0.0047	0.0134	-0.0049	0.0132	-0.0085	0.0126	0.0007	0.0126	-0.0024	0.0057
	(0.0115)	(0.0102)	(0.0107)	(0.0095)	(0.0108)	(0.0095)	(0.0110)	(0.0094)	(0.0098)	(0.0084)	(0.0085)	(0.0074)
Log out-degree × 16-19 years	-0.0183	0.0094	-0.0209	0.0076	-0.0213	0.0069	-0.0275*	0.0060	-0.0067	0.0084	-0.0057	0.0037
	(0.0147)	(0.0139)	(0.0147)	(0.0139)	(0.0147)	(0.0139)	(0.0146)	(0.0137)	(0.0142)	(0.0133)	(0.0152)	(0.0123)
Observations	2,376,934	2,314,960	2,376,934	2,314,960	2,376,934	2,314,960	2,376,934	2,314,960	2,376,934	2,314,960	2,376,934	2,314,960
R^2	0.19166	0.12289	0.20170	0.14325	0.20191	0.14564	0.20807	0.14643	0.31503	0.15959	0.32843	0.18239
Within R^2	0.02290	0.00656	0.02272	0.00635	0.02284	0.00658	0.02026	0.00606	0.00204	0.00216	0.00210	0.00260
Tenure group fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Female fixed effects			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Italian fixed effects					✓	✓	✓	✓	✓	✓	✓	✓
Past spells fixed effects							✓	✓	✓	✓	✓	✓
Starting date fixed effects									✓	✓	✓	✓
Province fixed effects											✓	✓

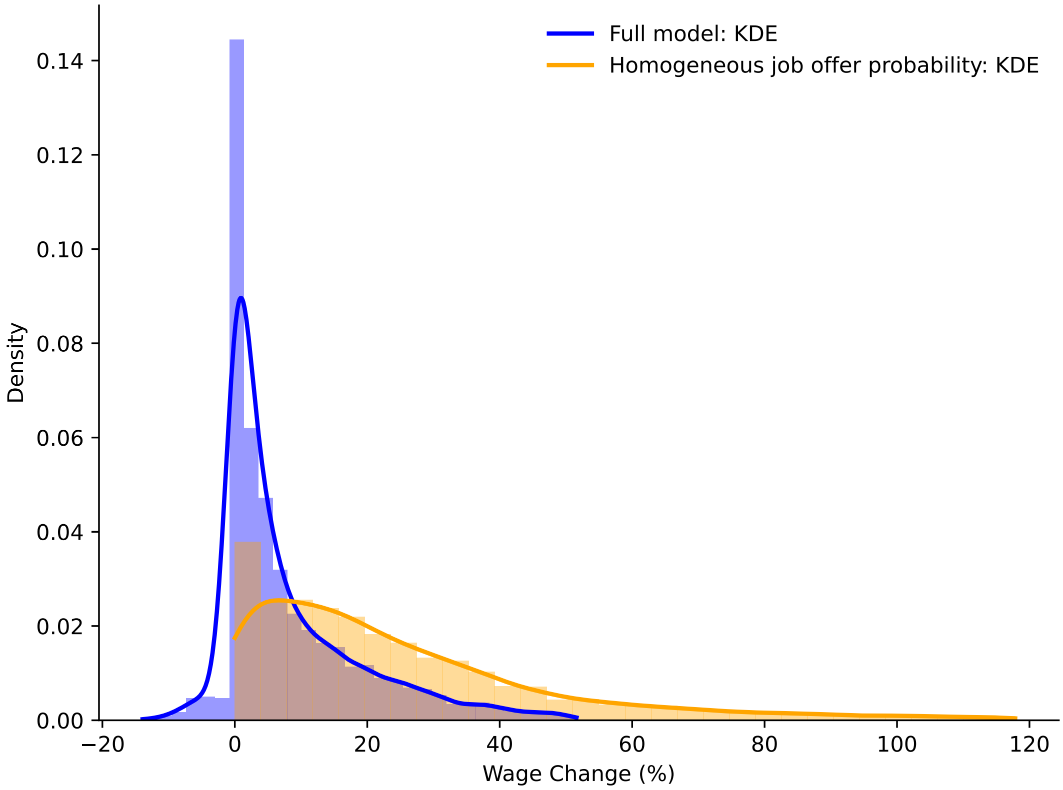
Notes. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Table A10 — Relationship between origin-firm AKM FE and per-worker out-degree centrality

	(1) Past firm FE $\times$ tenure > 2y	(2) Past firm FE $\times$ tenure ≤ 2y
Log per-worker out-degree	0.00569*	0.0104***
	(2.15)	(3.71)
Constant	-0.0968***	-0.0519***
	(-11.20)	(-5.89)
Observations	56720	60589

Notes. This table reports the coefficients of two separate OLS regressions of origin firm fixed effects interacted by firm-specific tenures on log per-worker out-degree centralities. The fixed effects are estimated as per model 14. Parenthesis report the t-statistics. $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$. Source: Istituto Nazionale della Previdenza Sociale (INPS).

Figure A3 — Distribution of wage changes: estimated model vs. counterfactual with homogeneous job offer probability



Notes. This figure displays the estimated kernel density distribution of wage changes for two scenarios: the estimated full model (in blue) and the counterfactual model where all employed workers have a uniform probability of receiving an outside job offer during on-the-job search.

B — Additional information on data cleaning

Here, I detail the data-cleaning operation undertaken on the firm panel and the matched employer-employee dataset.

Firm panel I start from the *Uniemens* database provided by INPS. The primary cleaning required by this firm-by-year panel is to assign a single province and four-digit industry for each observation—the same firm might indeed operate in multiple sectors or geographical areas within the same period. For both, I do so by imputing the observation with the highest number of employees in the year. Then, I restrict the sample to firms that have employed at least fifteen workers at least once over the 2006-2018 period.

Matched employer-employee I build the matched employer-employee dataset at the monthly level starting by appending separate yearly files. First, I restrict the sample of workers and firms. I keep only workers who have been employed in a white-collar position at least once over the reference period. Among these employees, I further restricted to those that have moved between large firms only—as previously defined. Within this subsample, I drop contracts that lasted less than nine weeks in a given year and contracts with zero wage, and I winsorize the wage outliers at the 0.45 and 99.5 percentiles. I also restrict the contract sample to full-time jobs. Then, I assign each worker to *one* firm with *one* contract each year. To do so, I need to solve for the occurrence of multiple spells, both *within* and *between* worker-firm pairs. When facing multiple spells in the same month within the same employer—i.e., two contemporaneous contracts within the same firm in a given period—I keep the one that pays more. Then, I resolve multiple spells across different employers within the same month through a nested criterion: I keep the one that involves more worked days and, subordinately, the one that pays more. Finally, I perform minor cleanings related to unreliable measures, such as dropping workers who have been paid more than 365 days per year and workers who entered the job market when younger than 18 or older than 50.

C — Additional degree measures

For complementary results and robustness checks, I rely on two additional centrality measures.

The first is the weighted degree centrality. Unlike its unweighted counterpart, which simply counts the number of links to a node, the weighted variant sums the weights—i.e., the intensity of the worker flow entering or leaving the firm. Formally, in an undirected scenario, it's expressed as

$$D^w(i) = \sum_{j}^{N} W_{ij}$$

where W represents the weighted adjacency matrix and W_{ij} the flow between i and j over a given period. This measure only accounts for a node's total involvement in the network, ignoring the number of other nodes it's connected to.

Therefore, I rely on Opsahl et al. (2010) to create a hybrid measure considering both a node's degree and strength. Specifically, the Opsahl centrality equals the number of connected nodes times the adjusted average weight of these nodes:

$$D^\alpha(i) = D(i)\left(\frac{D^w(i)}{D(i)}\right)^\alpha = D(i)^{1-\alpha} \cdot D^w(i)^\alpha$$

where $\alpha > 0$ is a tuning parameter fixing the importance of links quantity relative to their weight. Here, α is set to 0.5.

D — K-means

The k-means algorithm is an unsupervised clustering method that, *ex-ante*, only asks for the number of partitions to split the sample in. In this appendix, I discuss the choice of dividing the firms into two groups: high- and low-connectivity. Following Makles (2012), I involve four measures as an optimality criterion to infer the optimal number of clusters: the within sum of squares (WSS), its logarithm, the η^2 coefficient defined as

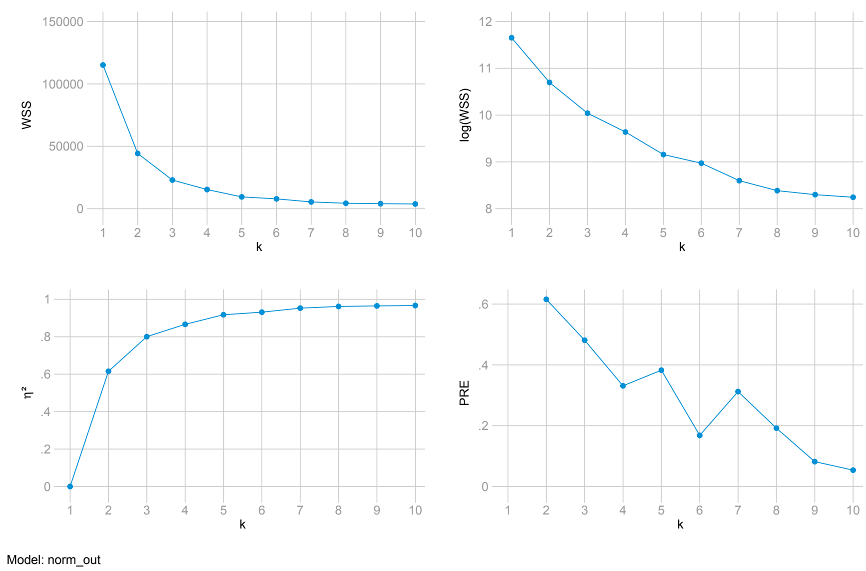
$$\eta_k^2 = 1 - \frac{\text{WSS}(k)}{\text{WSS}(1)} = 1 - \frac{\text{WSS}(k)}{\text{TSS}} \quad \forall k$$

where $\text{WSS}(k)$ is the WSS for a clustering with k partitions; and the proportional reduction error (PRE) given by

$$\text{PRE}_k = \frac{\text{WSS}(k-1) - \text{WSS}(k)}{\text{WSS}(k-1)} \quad \forall k \geq 2$$

Basically, the η_k^2 accounts for the proportional reduction of the WSS for each clustering with k partitions, compared with the total sum of squares (TSS). PRE_k measures the proportional reduction of the WSS for each added cluster.

Figure D1 — Optimal-splitting criteria for 1-to-10 clusters



Notes. The figure shows the optimal splitting criteria from 1 to 10 clusters. The highest gains in both relative and absolute terms is achieved for $k = 2$.

Figure D1 plots these four indicators computed on the firms' sample split on the normalized out-degree centrality. Splitting the sample in two is the best choice in terms of WSS and

$\log(\mathrm{WSS})$ reduction, as the two top panels in the figure show the deepest kink for $k = 2$. Moreover, η_2^2 records a 60% reduction in the WSS in absolute terms, while PRE_2 gives the highest gain in terms of proportional WSS decrease.

E — Theory

E.1 – Derivation of the wage equation

I derive the wage equation using the employed worker value function (4) and the unemployed worker value function (5), applying the bargaining protocol to obtain the joint surplus for all $(\theta, \hat{\theta})$.

Consider

$$U = u + \beta \left[\lambda \int_u \left(W\left(a, x, y, u_p, 0\right) - U \right) + U \right]$$

Then, the net of unemployment value for a worker is

$$\begin{aligned} W\left(a, \theta, \hat{\theta}\right) - U = w\left(a, \theta, \hat{\theta}\right) - u \\ + \beta \Bigg\{ (1-\delta) \Bigg[\int_{I-\theta_r}^{\bar{\theta_r}} \Bigg(\int_{x \in \mathcal{F}_1(\theta_p, y)} W\left(a, x, y, \theta\right) \, \mathrm{d}P(x) \\ + \int_{x \in \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} W(a, \theta, x, y) \, \mathrm{d}P(x) \Bigg) \, \mathrm{d}T(y) \\ + \left(1 - \int_{I-\theta_r}^{\bar{\theta_r}} \int_{x \in \mathcal{F}_1 \cup \mathcal{F}_2} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) W\left(a, \theta, \hat{\theta}\right) - U \Bigg] \\ - \lambda \int_u \left(W\left(a, x, y, u_p, 0\right) - U \right) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \Bigg\} \end{aligned}$$

Since the sets are disjoint, it is possible to write

$$\begin{aligned} W\left(a, \theta, \hat{\theta}\right) - U = w\left(a, \theta, \hat{\theta}\right) - u \\ + \beta \Bigg\{ (1-\delta) \Bigg[\int_{I-\theta_r}^{\bar{\theta_r}} \Bigg(\int_{x \in \mathcal{F}_1(\theta_p, y)} \left[W\left(a, x, y, \theta\right) - U \right] \, \mathrm{d}P(x) \\ + \int_{x \in \mathcal{F}_2\left(\theta_p, y, \hat{\boldsymbol{\theta}}\right)} \left[W(a, \theta, x, y) - U \right] \, \mathrm{d}P(x) \Bigg) \, \mathrm{d}T(y) \\ + \left(1 - \int_{I-\theta_r}^{\bar{\theta_r}} \int_{x \in \mathcal{F}_1 \cup \mathcal{F}_2} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) \left[W\left(a, \theta, \hat{\theta}\right) - U \right] \Bigg] \\ - \lambda \int_u \left(W\left(a, x, y, u_p, 0\right) - U \right) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \Bigg\} \end{aligned}$$

Applying the bargaining protocol leads to

$$
\begin{aligned}
W\left(a,\theta,\hat{\theta}\right)-U=&\;w\left(a,\theta,\hat{\theta}\right)-u\\
&+\beta\left\{(1-\delta)\left[\int_{I-\theta_r}^{\bar{\theta_r}}\left(\int_{x\in\mathcal{F}_1(\theta_p,y)}\left[(1-\sigma)S\left(a,\theta\right)+\sigma S\left(a,x,y\right)\right]\,\mathrm{d}P(x)\right.\right.\right.\\
&+\left.\int_{x\in\mathcal{F}_2\left(\theta_p,y,\hat{\boldsymbol{\theta}}\right)}\left[(1-\sigma)S\left(a,x,y\right)+\sigma S\left(a,\theta\right)\right]\,\mathrm{d}P(x)\right)\,\mathrm{d}T(y)\\
&+\left.\left(1-\int_{I-\theta_r}^{\bar{\theta_r}}\int_{x\in\mathcal{F}_1\cup\mathcal{F}_2}\mathrm{d}P(x)\,\mathrm{d}T(y)\right)\left[W\left(a,\theta,\hat{\theta}\right)-U\right]\right]\\
&\left.-\lambda\sigma\int_u S\left(a,x,y\right)\,\mathrm{d}P(x)\,\mathrm{d}T(y)\right\}
\end{aligned}
\tag{E1}
$$

By noting that $W\left(a,\theta,\hat{\theta}\right)-U=(1-\sigma)S\left(a,\hat{\theta}\right)+\sigma S\left(a,\theta\right)$, we can write (E1) as

$$
\begin{aligned}
[1-\beta(1-\delta)]\left[W\left(a,\theta,\hat{\theta}\right)-U\right]=&\;w\left(a,\theta,\hat{\theta}\right)-u\\
&+\beta\left\{(1-\delta)\left[\int_{I-\theta_r}^{\bar{\theta_r}}\left(\int_{x\in\mathcal{F}_1(\theta_p,y)}\left[(1-\sigma)\left[S\left(a,\theta\right)-S\left(a,\hat{\theta}\right)\right]\right.\right.\right.\right.\\
&+\left.\sigma\left[S\left(a,x,y\right)-S\left(a,\theta\right)\right]\right]\,\mathrm{d}P(x)\\
&+\left.\left.(1-\sigma)\int_{x\in\mathcal{F}_2\left(\theta_p,y,\hat{\boldsymbol{\theta}}\right)}\left[S\left(a,x,y\right)-S\left(a,\hat{\theta}\right)\right]\,\mathrm{d}P(x)\right)\,\mathrm{d}T(y)\right]\\
&\left.-\lambda\sigma\int_u S\left(a,x,y\right)\,\mathrm{d}P(x)\,\mathrm{d}T(y)\right\}
\end{aligned}
\tag{E2}
$$

Now define

$$\eta\equiv u+\beta\lambda\sigma\int_u S\left(a,x,y\right)\,\mathrm{d}P(x)\,\mathrm{d}T(y)$$

so we can write (E2) as

$$W\left(a,\theta,\hat{\theta}\right)-U=\frac{w+\beta\left(1-\delta\right)G\left(a,\theta,\hat{\theta}\right)-\eta}{1-\beta\left(1-\delta\right)}$$

where $G\left(a,\theta,\hat{\theta}\right)$ collects the gains from on-the-job search for the worker:

$$
\begin{aligned}
G\left(a,\theta,\hat{\theta}\right) = \int_{I-\theta_r}^{\bar{\theta_r}} \Bigg(\int_{x\in\mathcal{F}_1(\theta_p,y)} & \Big[(1-\sigma)\left[S\left(a,\theta\right) - S\left(a,\hat{\theta}\right)\right] \\
& + \sigma\left[S\left(a,x,y\right) - S\left(a,\theta\right)\right]\Big] \,\mathrm{d}P(x) \\
& + (1-\sigma)\int_{x\in\mathcal{F}_2\left(\theta_p,y,\hat{\boldsymbol{\theta}}\right)} \left[S\left(a,x,y\right) - S\left(a,\hat{\theta}\right)\right] \,\mathrm{d}P(x)\Bigg) \,\mathrm{d}T(y)
\end{aligned}
$$

Noticing that

$$
(1-\sigma)S\left(a,\hat{\theta}\right) + \sigma S\left(a,\theta\right) = \frac{w + \beta\left(1-\delta\right)G\left(a,\theta,\hat{\theta}\right) - \eta}{1-\beta(1-\delta)}
$$

and going back to the surplus definition appropriately manipulated

$$
\begin{aligned}
\left[1-\beta(1-\delta\right] S\left(a,\theta\right) = f\left(a,\theta_p\right) \\
+ \beta\left(1-\delta\right)\sigma \int_{I-\theta_r}^{\bar{\theta_r}} \int_{x\in\mathcal{F}_1(\theta_p,y)} \left[S\left(a,x,y\right) - S\left(a,\theta\right)\right] \,\mathrm{d}P(x)\,\mathrm{d}T(y) - \eta
\end{aligned}
$$

it is possible to write

$$
\begin{aligned}
[1-\beta(1-\delta)](1-\sigma)S\left(a,\hat{\theta}\right) + \sigma f\left(a,\theta_p\right) + \beta(1-\delta)\sigma^2 \int_{I-\theta_r}^{\bar{\theta_r}} \int_{x\in\mathcal{F}_1(\theta_p,y)} \left[S\left(a,x,y\right) - S\left(a,\theta\right)\right] \\
= w\left(a,\theta,\hat{\theta}\right) + \beta(1-\delta)G\left(a,\theta,\hat{\theta}\right) - \eta
\end{aligned}
$$

from which one easily gets Equation (8).

E.2 – Comparative statics for the wage equation

With the wage equation, it is possible to operate a comparative statics exercise on wages concerning the two dimensions of firms' heterogeneity. First, define $\Pi(\theta,\hat{\theta})$ as

$$
\begin{aligned}
\Pi(\theta,\hat{\theta}) \equiv \int_{I-\theta_c}^{\bar{\theta}_c} \Bigg(\int_{x\in\mathcal{F}_1(\theta_p,y)} & \overbrace{\left(\sigma^2-\sigma\right)\left[S(x,y) - S(\theta)\right]}^{<0} \,\mathrm{d}P(x) \\
- \underbrace{(1-\sigma)\left[S(\theta) - S(\hat{\theta})\right]}_{>0} - & \int_{x\in\mathcal{F}_2\left(\theta_p,y,\hat{\boldsymbol{\theta}}\right)} \underbrace{(1-\sigma)\left[S(x,y) - S(\hat{\theta})\right]}_{>0} \,\mathrm{d}P(x)\Bigg) \,\mathrm{d}T(y) < 0
\end{aligned}
$$

Then, one has

$$\frac{\partial w(a,\theta,\hat{\theta})}{\partial \theta_c} = \frac{\partial \Pi(\theta,\hat{\theta})}{\partial \theta_c} < 0$$

since the surplus is strictly increasing in θ_c.

When considering θ_p, one has

$$\frac{\partial w(a,\theta,\hat{\theta})}{\partial \theta_p} = \sigma a + \frac{\partial \Pi(\theta,\hat{\theta})}{\partial \theta_p}$$

Therefore, the sign of the derivative depends on α. In particular, when $\sigma = 0$, $\partial w/\partial \theta_p < 0$; when $\sigma = 1$, the opposite.

E.3 – Proof of Proposition 1

Proof. Let A represent a node in the network G—specifically, a firm—with a connectivity parameter c_A. The search process in the model can be divided into two stages at any given time. The first concerns meetings between firms, and the second involves formalizing the offer to the worker and potentially establishing a link. As the first step occurs with a constant, uniform probability independent of firms' connectivity, after a sufficient number of iterations, each firm (node) will eventually connect with the mass of all possible nodes to which it can link, i.e., $1 - T(I - c_A)$. This constitutes the expected relative degree centrality for an infinite number of iterations and is a function that increases with c_A. Assuming a large enough number of iterations for convergence completes the proof. □

E.4 – Solution to the planning problem

The utilitarian planner's problem is straightforward. The planner determines which jobs are suitable for the unemployed and which are preferable for the employed. The objective is to maximize the expected present value of a worker's flow output produced by a worker in a match.

Let $Y_P(a, \boldsymbol{\theta})$ represent the present discounted value of output produced by a worker of ability a employed in a firm $\boldsymbol{\theta} = (\theta_p, \theta_c)$. Denote with $\mathcal{F}_P(\boldsymbol{\theta})$ the set of firms yielding a higher social net value than (θ). This set identifies the firms to which a planner would move a worker currently employed at $\boldsymbol{\theta}$. Let U_P be the present discounted value of output generated by an unemployed worker, who accepts a job offer from $\boldsymbol{\theta}$ if it falls in the set $\mathcal{F}_P(u)$ of firms exceeding the planner's reservation margin. Lastly, define $S_P(a, \boldsymbol{\theta}) \equiv \max\{0, Y_P(a, \boldsymbol{\theta}) - U_P(a, \boldsymbol{\theta})\}$

as the social net value of an employed worker. Then, we have

$$
\begin{aligned}
Y_P(a, \boldsymbol{\theta}) = f(a, \theta_p) + \beta \Bigg\{ (1-\delta) \Bigg[\int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_P(\theta_p, y)} Y_P(a, x, y) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \\
+ \left(1 - \int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_P(\theta_p, y)} \mathrm{d}P(x) \, \mathrm{d}T(y) \right) \max \left\{ Y_P(a, \boldsymbol{\theta}), U_P \right\} \Bigg] + \delta U_P \Bigg\}
\end{aligned}
$$

$$
U_P = u + \beta \left[\lambda \iint_{x, y \in \mathcal{F}_P(u)} \max \left\{ Y_P(a, \boldsymbol{\theta}), U_P \right\} \, \mathrm{d}P(x) \, \mathrm{d}T(y) + \delta U \right]
$$

$$
\begin{aligned}
S_P(a, \boldsymbol{\theta}) = \max \Bigg\{ 0, f(a, \theta_p) - u + \beta \Bigg[(1-\delta) \Bigg(S_P(a, \boldsymbol{\theta}) \\
+ \int_{I-\theta_c}^{\bar{\theta}_c} \int_{x \in \mathcal{F}_P(\theta_p, y)} \left[S_P(a, x, y) - S_P(a, \boldsymbol{\theta}) \right] \, \mathrm{d}P(x) \, \mathrm{d}T(y) \Bigg) \\
- \lambda \iint_{x, y \in \mathcal{F}_P(u)} S_P(a, x, y) \, \mathrm{d}P(x) \, \mathrm{d}T(y) \Bigg] \Bigg\}
\end{aligned}
$$

Solving for $S_P(a, \boldsymbol{\theta})$ provides the characterization of sets $\mathcal{F}_P(\boldsymbol{\theta})$ and $\mathcal{F}_P(u)$, which constitutes the solution to the planner's problem By noticing that

$$
S_P(a, \boldsymbol{\theta}) = S(a, \boldsymbol{\theta}) \quad \text{iff } \sigma = 1
$$

it becomes evident that the socially efficient ranking of jobs and reservation strategies can be directly obtained by solving the equilibrium value functions for joint surplus and unemployment when $\sigma = 1$.

F — Details on the model's estimation

The parameters of the model are estimated through a two-step process, which combines simulation-based methods with machine learning techniques to facilitate global optimization.

Objective Function. The model parameters are estimated by solving the following optimization problem:

$$\hat{\phi} = \arg\min_{\phi} \left\{ ((\phi_0) - \tilde{}(\phi))' \, W \, ((\phi_0) - \tilde{}(\phi)) \right\} \tag{F1}$$

where $\hat{\phi}$ is the $K \times 1$ vector of estimated parameters, (ϕ_0) is the $N \times 1$ vector of data-derived moments with true parameter values ϕ_0,$\tilde{}$is the simulated version, and W is a suitable weighting matrix.[15] Here, K represents the number of parameters of interest, and N denotes the number of targeted moments. The estimation targets the relative difference between $N = 13$ real and simulated quantities containing moments and reduced-form coefficients, to estimate $K = 9$ parameters (see Table 1).

Preliminary Simulation and Neural Network Training. As a first step, I build a surrogate model to explore the state space easily. I first generate a Sobol sequence of 4,096 parameter draws and simulate the model for these values. This yields a mapping of parameters to moments, which is used to train a small neural network to guide the subsequent exploration of the state space. Formally, let the map $f : \Phi \to \mathcal{M}$ represent this relationship, where Φ is the parameter space and $\mathcal{M}$ is the space of simulated moments and reduced-form parameters. The neural network $N(\cdot, \omega)$ aims to approximate f by solving

$$\omega^* = \arg\min_{\omega} \mathbb{E}_{\phi \sim P(\Phi)} \left[\| N(\phi, \omega) - f(\phi) \|^2 \right]$$

where $P(\Phi)$ denotes the distribution over the model parameters and $\| \cdot \|$ is the L-2 norm. This surrogate model enables the efficient identification of promising regions in the state space through 500,000 simulations that use the network rather than the expensive whole model.

This surrogate model allows me to grasp a broad idea of how the state space is composed without requiring a huge amount of simulations, which would take a lot of time and computational resources. I indeed simulate the surrogate model 500,000 times and observe where

15. To prevent the issue of heterogeneous moments weighting, I use a $K \times K$ identity matrix as a consistent yet straightforward choice.

minima are hit.

Bayesian optimization. Finally, I employ a Bayesian Adaptive Direct Search (BADS) algorithm (Acerbi and Ma, 2017) for focused exploration, starting from the ten best-performing parameter sets identified by the surrogate model $N(\phi, \omega^*)$. The algorithm alternates between poll steps and search steps. During the poll stage, steps are taken in one direction at a time on a mesh of the parameter's space, doubling the step size on success and halving it otherwise. In the search stage, a Gaussian process (GP) is fitted to a local subset of evaluated points. The algorithm iteratively selects parameters based on a lower confidence bound strategy, balancing between exploring uncertain regions (high GP uncertainty) and exploiting promising areas (low GP mean).

The estimated parameters vector $\hat{\phi}$ is then picked as the global minimum of these ten minimizations.